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# Dynamical ordering induced by preferential transitions in planar arrays of superheated superconducting granules

A. Peñaranda\*, L. Ramírez-Piscina

*Departament de Física Aplicada, Universitat Politècnica de Catalunya, Avda. Gregorio Marañón 44, E-08028 Barcelona, Spain*

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## Abstract

We perform simulations of planar arrays of superheated superconducting granules (PASS) under an external magnetic field, analyzing transitions undergone by the system when the external field is slowly increased from zero. We observe, for high concentrations, the existence of an interval of external fields for which no transitions are induced. This effect is analogous to a 'hot border zone' identified in the response of superheated superconducting granule detectors. We explain such behaviour as produced by a geometrical ordering dynamically induced in the system by transitions in preferential sites due to diamagnetic interactions.

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Superheated superconducting granules systems are being developed as detectors in areas such as dark matter, neutrino, neutron, X-ray and transition radiation [1]. An ensemble of spherical granules of a Type I superconductor material is maintained in a metastable superheated state by adequate conditions of temperature and external magnetic field. An increasing in the applied field or the deposition of energy in a microgranule by radiation can produce a transition to the normal state. The change of magnetic flux inherent to the loss of Meissner effect can be sensed by a sensitive magnetometer which provides information about the incident radiation.

An early detector was proposed as a disordered colloidal suspension of microgranules in a suitable medium such as paraffin wax [2]. The state of each grain of this suspension in the phase diagram depends, in addition to the external field and temperature, on its metallurgical defects and the diamagnetic interactions of the other grains of the suspension. Metallurgical defects can increase the local magnetic

field of the grain and can precipitate the transition to normal state. Diamagnetic interactions depend on the environment of each grain, producing an additional dispersion in the surface magnetic field values. As a consequence of these combined effects, the spread of transitions fields of the suspension reaches values of about 20% that can reduce the resolution of the detector [3].

In a previous work we showed that this spreading in transition fields is effectively reduced following an increase in the applied field. We obtained that the successive transitions induced by the external field are a strong ordering mechanism, which produce a more homogeneous distribution of surface magnetic fields of the granules. Consequently, by using this effect, the uncertainty could be reduced in these devices [4,5].

Experimentally, a variant of the colloidal device has been developed in response to the spread problem [6]. The microgranules are arrayed on thin planar substrates. These planar arrays of superconducting spheres (PASS) have yielded differential superheating curves in which the spreading is reduced by an order of magnitude.

Although the technique of fabrication of this device can only produce arrays of relative small size, it has been shown

\* Corresponding author. Tel.: +34-93-4017995; fax: +34-93-4017996.

E-mail address: [angelina@fa.upc.es](mailto:angelina@fa.upc.es) (A. Peñaranda).

that the PASS has both good energy and position sensitivity. On the other hand, the avalanche effect, demonstrated in lines of granules, can enhance the magnetic signal. This allows, in principle, the use of very small grains in devices with high energy sensitivity [7]. We presented in a previous work [8] simulations of these systems, and results of maximum surface fields were shown for different distances between spheres, i.e. different concentrations. The broadening of maximum surface field distributions for increasing concentrations as a consequence of diamagnetic interactions, with noticeable finite size effects, was noted. This would produce a larger dispersion in transition fields.

Numerical Monte Carlo techniques were applied by Esteve et al. [9] to study transitions induced in a bidimensional lattice of small superconducting spheres by an external magnetic field normally applied to the array. Even though they only considered two body interactions and diamagnetic contributions until a small number of neighbours of each sphere, they found an interesting phenomenon: a gap or plateau zone appears in the curves of transition counts versus the applied field for systems with small lattice spacing, which seems always located around a fraction  $f$  of remaining superconducting spheres equal to 0.30. At this point, a large increase of field is required to observe the next transition. The qualitative explanation given for the existence of the plateau is the decreasing of diamagnetic interactions due to the change in distance between spheres as a result of transitions. Nevertheless the authors did not provide any explanation of the observed discontinuity nor the  $f = 0.30$  occurrence of this phenomenon.

This plateau corresponds to an effective ‘hot border zone’ previously observed in the response of superheated superconducting grains devices, in the sense that the system is insensitive to small changes in external field, and transitions can only be produced by thermal nucleation [10]. This effect has interesting consequences in PASS operations, because it introduces a threshold in the value of the energy of the incident radiation to permit transitions to occur.

In this paper we perform numerical simulations of transitions in PASS configurations immersed in an increasing external field,  $B_{\text{ext}}$ , in order to analyse this phenomenon. We observe that the existence or not of the plateau is related to different dynamical evolutions, for concentrated or dilute systems, during transitions. This dynamics produces, in the case of more concentrated systems, a spatial order coherent with the appearance of the plateau zone at about  $f = 0.25$  value. This ordering is the result of previous preferential transitions in certain lattice sites due to diamagnetic interactions. However, transitions in dilute systems give an evolution to a different kind of spatial configuration. A key point in this differentiation is the role of diamagnetic interactions, which are stronger in more concentrated systems.

We consider the applied magnetic field  $B_{\text{ext}}$  perpendicu-

lar to the planar system. We assume the microgranules as spheres of equal radius  $a$  much larger than the London penetration length. We consider that the transition of each microgranule to the normal phase is completed once the local magnetic field at any point of its surface reaches a threshold value  $B_{\text{th}}$ . This value can vary from sphere to sphere and is introduced in order to take into account the defects of the spheres.

We employ, in our simulations, a distribution of threshold values experimentally determined for tin microspheres dispersed in paraffin. This distribution was fitted by a parabolic distribution in a range of values between  $B_{\text{SH}}(1 - \Delta)$  and  $B_{\text{SH}}$  [11]. Small values of  $\Delta$  are related to more perfect spheres. Most results shown in this work correspond to  $\Delta = 0.2$ .

The procedure in our simulations is as follows:  $N$  spheres are placed in a square array, separated a distance  $d$ . After assignment at random of a threshold field value to each sphere, the system is immersed in an external magnetic field  $B_{\text{ext}}$  which is slowly increased from zero. The knowledge of the surface magnetic field on each microsphere is achieved by solving the Laplace equation with the appropriate boundary conditions. We have used a numerical procedure that allows us both to consider the complete multi-body problem and to reach multi-polar contributions of arbitrary order [8]. When the maximum local magnetic field on the surface of any sphere reaches its threshold value  $B_{\text{th}}$ , the sphere transits and the configuration becomes one of  $N - 1$  superconducting spheres. The change of diamagnetic interactions in any transition leads us to repeat the process until all spheres have transited. The maximum surface magnetic field value of each sphere is monitored after each transition, allowing us to study the evolution of a system in its successive transitions.

Numerical simulations have been performed on several configurations with distances between sphere centers, in units of radius  $a$ , of  $d/a = 7.482, 4.376, 3.473, 3.034, 2.757$  and 2.5. These distances correspond in a 3D array to values of filling factor (fraction of volume occupied by the spheres), of  $\rho = 0.01, 0.05, 0.10, 0.15, 0.20$  and 0.268.

An important point to be considered in this kind of system is the finite-size effects [8] that affect surface magnetic field values, especially in dense configurations, and that force us to work with a number of microgranules as large as possible [8]. By computational limits and precision requirements, the number of spheres analysed has been  $N = 400$  in the more concentrated systems and  $N = 169$  in the other configurations.

Results of simulations of field-induced transitions are shown in Fig. 1. In this figure, the fraction  $f$  of still superconducting spheres versus the (increasing) external field, referred to the critical superheating field ( $B_{\text{ext}}/B_{\text{SH}}$ ), is presented for several values of lattice spacing. We can observe a fast decay in the most dilute case, in which transitions are produced for external field values closely following the distribution of threshold values. Transitions

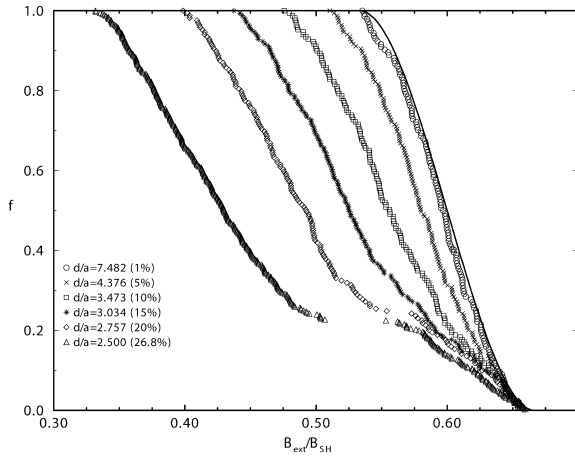


Fig. 1. Fraction  $f$  of still superconducting spheres versus  $B_{\text{ext}}/B_{\text{SH}}$ , after an increase of the perpendicular external magnetic field from zero, for several samples of  $N = 169$  initially superconducting spheres, corresponding to different initial lattice spacings. ( $N = 400$  for the more concentrated systems). Continuous line corresponds to the dilute limit, i.e. assuming a maximum surface field of  $3/2B_{\text{ext}}$  for all the spheres.

begin for smaller external fields values as the concentration of the system increases. This shows the significance of diamagnetic interactions on local surface field values, which is stronger for spheres in closer proximity. On the other hand, the transition curves spread out for these concentrated configurations. But the more significant effect shown in this figure is the breakdown of the continuous response and the appearance of a ‘plateau zone’ clearly distinguished for shorter lattice spacing. This effect is produced for a fraction of remaining superconducting spheres slightly lower than  $f = 0.25$ . In this zone there is a gap in the necessary increment of the external field to generate the following transition. The width of this gap increases as the lattice spacing is reduced. This plateau corresponds to an effective ‘hot border’.

Comparison with results from the work of Esteve et al. [9], shows great similarity even though the location of the plateau is in their case always around  $f = 0.3$ . They worked with perfect spheres and two-body interactions, which were only considered for spheres closer than a few lattice spacing. They interpreted this zone as an interpolation between two qualitatively different dilute regimes. One would correspond to a initially homogeneous system, for large values of  $d/a$ , where diamagnetic interactions are not very important. The other corresponds to a regular configuration obtained as a consequence of the dilution of an initially concentrated system after transitions.

We analyse this effect by studying the dynamics of the system in its evolution during the increase of the external field. We consider both the spatial configurations and the distributions of surface fields that change after each transition. Some of our results are represented in Fig. 2,

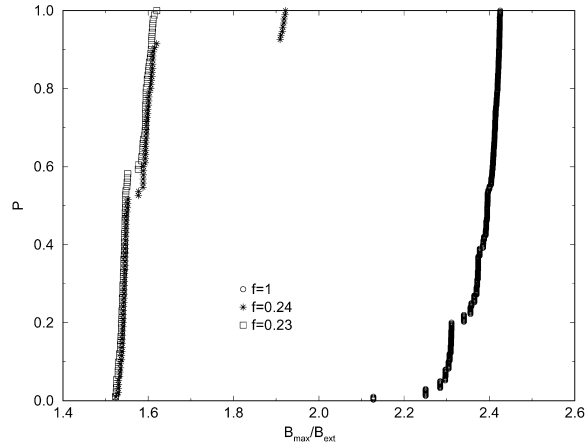


Fig. 2. Fraction  $P$  of spheres with maximum surface field lower than the x-axis value (in units of  $B_{\text{ext}}$ ), in the evolution of a configuration with initial lattice spacing  $d/a = 2.5$  ( $\rho = 0.268$ ) and  $N = 400$  near the plateau zone.

where the maximum surface magnetic field distributions for a configuration with  $d/a = 2.5$  are shown at three values of the increasing external field. Namely we present distributions for the initial state ( $f = 1$ , all 400 granules are still superconducting) and for configurations before and after the plateau ( $f = 0.24$  and  $f = 0.225$ , 96 and 90 superconducting spheres, respectively). Transitions induced by the external magnetic field split the initial distribution in two branches separated by a gap. When the system is reaching the plateau zone, only a small number of spheres are in the branch of high surface magnetic fields. Some of these microgranules will be the next to transit. Each transition affects the interactions between microgranules, especially in the nearest neighbours by reducing their surface field. This situation is reflected in Fig. 2 by the jump of each sphere from one branch to the other. The disappearance of the high field branch corresponds to the plateau zone. The remaining superconducting spheres have lower maximum surface fields, and need larger external fields to achieve their threshold value and turn to the normal state. This explains the presence of the plateau.

Looking closely at the spatial distributions, we observe that the branch with larger surface magnetic fields corresponds to spheres having a superconducting next neighbour, and hence experiencing stronger diamagnetic interactions. In the plateau zone, only spheres without superconducting next neighbours remain superconducting. The system reaches a quite regular configuration with a fraction of superconducting spheres of about  $f = 0.25$ . This is clearly shown in Fig. 3. In this figure a snapshot of positions of superconducting microgranules are represented just before and after the plateau zone.

An interesting question is how the dynamics of transitions leads the system to such ordered configurations, and why this occurs for higher concentrations and not for

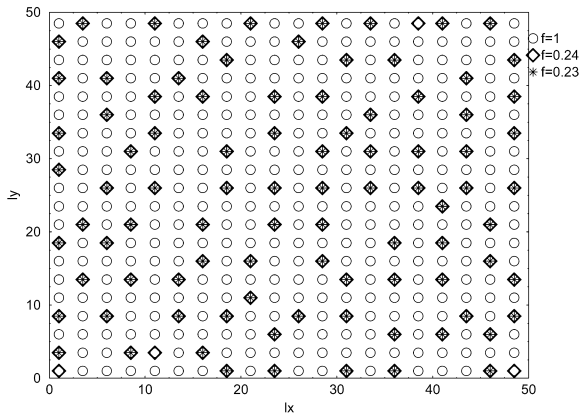


Fig. 3. Spatial distribution of initial  $N = 400$  spheres with lattice distance  $d/a = 2.5$  ( $\circ$ ), and the still superconducting spheres just before ( $\diamond$ ) and after ( $\star$ ) the plateau zone.

dilute systems. In order to gain insight into this phenomenon, we have studied in detail the dynamics of transitions in systems with a reduced dispersion of threshold fields (i.e. more perfect granules) and a larger number of initially superconducting spheres (in order to reduce finite-size and boundary effects). These systems show a more perfect spatial ordering at the plateau (with  $f$  very close to 0.25) and therefore are more suitable to analyse regarding spatial configurations during transitions. This study reveals an interesting behaviour. These more perfect spheres present two clearly different spatial distributions before reaching the plateau depending on the concentration (and corresponding to the appearance or not of the plateau at  $f = 0.25$ ). This is shown in Fig. 4 where the two different behaviours are compared at  $f = 0.5$  on systems with initial spheres separated distances  $d/a = 2.5$  and 3.034 (3D filling factor

$\rho = 26.8$  and 15%). For smaller distances between spheres (more concentrated systems, Fig. 4(b)), the remaining superconductor spheres, after a number of transitions, show a configuration separated into domains. In each of these domains, transitions are produced in such a way that spheres have a tendency to form parallel lines in a sort of ‘striped’ configuration. Until  $f = 0.5$  only spheres between lines transit. The resulting configuration is formed by alternately superconducting and normal lines. For more diluted systems (Fig. 4(a)) this patterning does not exist. Subsequent transitions in the concentrated systems are produced in such a way that in each line, transitions of granules occur with superconducting next neighbours. When the plateau appears, only spheres with third neighbours remain superconducting, forming a square lattice of spacing  $2d$ . This corresponds to a value of  $f = 0.25$  for the appearance of the plateau. In the systems with less perfect granules, the domains are smaller and not so clearly defined, and the plateau can appear for slightly smaller values of  $f$  due to more important boundary effects between domains.

From this dynamical study we observe that the ordered spatial configurations for  $f = 0.5$ , would condition the existence or not of the plateau at  $f = 0.25$ . We have elaborated a criterion that allows one to know if the striped configuration is possible, at  $f = 0.5$ , for a particular system. This criterion uses the simulation of a system representing one of these domains. We prepare this system with parallel stripes of superconducting granules and an additional granule in the middle. This additional granule should be the first to transit in order to reach the striped configuration. If it should not be so, this configuration could not be possible and then the plateau would not appear. We have performed simulations on one of these domains by placing  $N = 82$  spheres distributed in 9 lines of 9 spheres each, and the additional sphere in a central position between two lines.

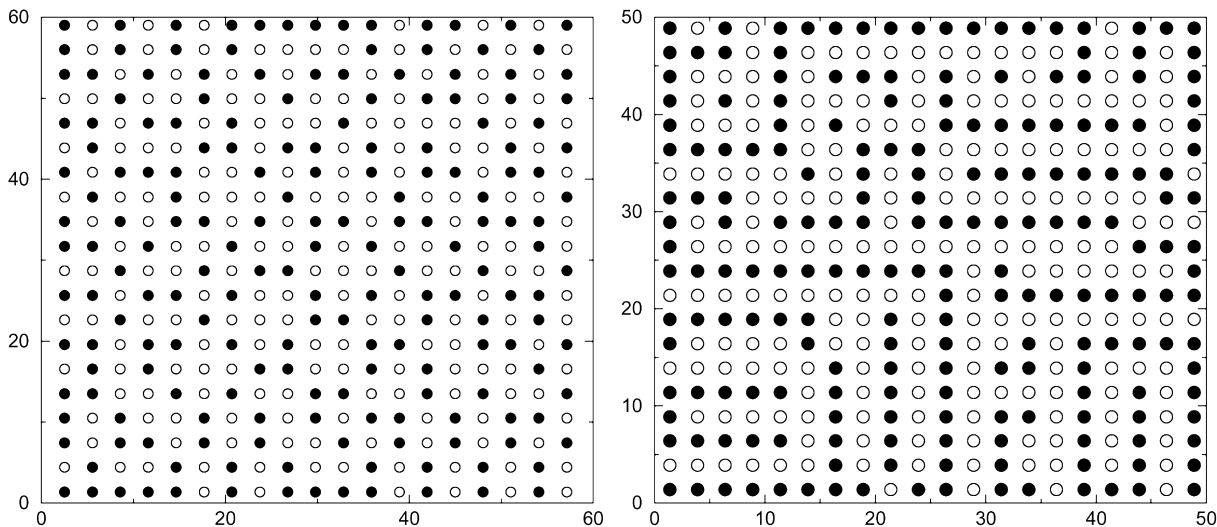


Fig. 4. Spatial distribution of initial  $N = 400$  spheres with lattice distance  $d/a = 3.034$  and 2.50 (3D 15 and 26.8%, respectively) and the corresponding distribution when half of the microspheres have transited ( $f = 0.5$ ).

Each line is separated by a distance of  $2d$  from the other. The distance between spheres of the same line is  $d$ . A diagram of this system is shown in the inset of Fig. 5. Analysing the maximum surface fields of the spheres in this configuration, for different lattice values, we have observed that for diluted systems, the sphere that has the highest maximum surface field (and that will be the next sphere to transit) is not the additional one. On the contrary, for more concentrated systems, the highest maximum surface field does correspond to the central sphere, and consequently this sphere will be the first to pass to the normal state. In this case, the remaining spatial configuration will be formed by complete lines. This is shown in Fig. 5 where the maximum surface fields of the spheres on the horizontal line containing the central sphere are presented for two representative values of  $d/a$ .

Repeating simulations for different values of the lattice distances permits a location of the limit between both behaviours, and consequently the density above which the plateau zone appears. We have obtained this limit for a lattice distance  $d/a = 2.871$  ( $\rho = 17.7\%$ ) in these ideal conditions. Results are shown in Fig. 6 for lattice distances near to this concentration. In this figure the maximum surface field of the central sphere and that of its next neighbour are compared.

It can be interesting to relate the response of the system to the applied field with the position of the ensemble of spheres in the phase diagram. In both concentrated and dilute systems, the first sphere to transit will be that with largest diamagnetic interactions and consequently with the highest maximum surface field (related to their threshold limit). In dilute systems, diamagnetic interactions are weak, the maximum surface field values of the spheres have a small dispersion and the population of still-superconducting spheres will present a quasi-continuous distribution in the

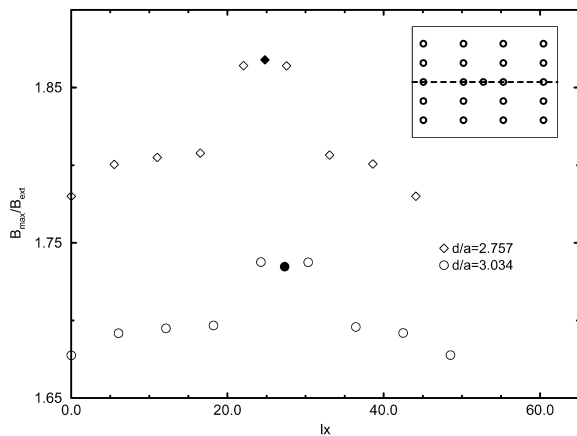


Fig. 5. Maximum surface magnetic field (in units of  $B_{ext}$ ) for spheres, in a striped domain, with spatial configuration represented in the inset of the figure. The field values on spheres lying on a line containing the central sphere are represented for  $d/a = 3.034$  and  $2.757$  ( $\rho = 0.15$  and  $0.20$ ).

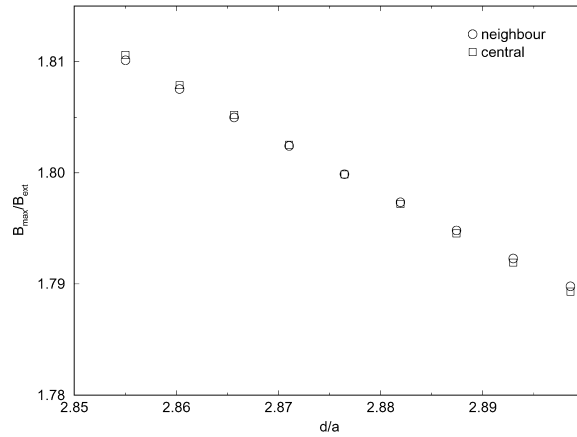


Fig. 6. Maximum surface magnetic field (in units of  $B_{ext}$ ), versus lattice distances, corresponding to the central sphere and its neighbour of a striped domain.

phase diagram. Small changes of applied field can produce subsequent flips to the normal state, and the transition curves present a continuous aspect. For more concentrated systems, the effect of diamagnetic interactions is very important. After successive transitions, the pairs of nearest superconducting spheres have higher surface fields, in comparison with those without superconducting next neighbours, as can be seen in Fig. 2. This effect separates the population of still superconducting spheres in two distinguished groups in the phase diagram, one, corresponding to spheres with higher surface fields and near the superheated line, and separated from the other corresponding to spheres with smaller values. Successive transitions change the population of each group. When the plateau appears, only the group of smaller field values remains superconducting. A small increment of the external field is unable to produce any transition; this is possible only by thermal nucleation. A larger increment of  $B_{ext}$  is necessary to continue the transitions. This is reflected as a ‘gap’, i.e. a plateau zone.

We can conclude that diamagnetic interactions play an important role in these kinds of systems, inducing distinct behaviours depending on their concentration. In the case of small lattice distances, a gap or plateau zone appears in the transition curves for a fraction of the remaining superconducting spheres of about  $f = 0.25$ . This plateau is a consequence of a spatial order achieved through preferential transitions in these concentrated configurations. This order produces a uniform distribution of surface magnetic field values, which is reflected in the phase diagram as a distribution of the population of still superconducting spheres separated from the superheated line. In this zone only transitions by finite increments of temperature are possible. This corresponds to a hot border. Transitions undergone by dilute systems follow different spatial distributions that do not bring the plateau appearance. From simulation of ideal systems of quasi-perfect spheres,

we have located the limit between the two behaviours at a lattice distance of  $d/a = 2.871$ .

Finally, it is worth to note the interest that this plateau has for a PASS detector, because the uncertainty in the energy threshold for transitions can be reduced in the presence of a hot border.

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