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TRANSITIONS IN DISORDERED SUSPENSIONS OF SUPERCONDUCTING GRANULES UNDER EXTERNAL MAGNETIC FIELD

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We perform simulations of transitions in three-dimensional suspensions of superconducting granules, placed completely at random in a sample, when an external magnetic field is slowly increased. The results show that these transitions induce order in the system. This effect appears as advantageous in applications for Superheated Superconducting Granule detectors.

Keywords: A. superconductors, A. disordered systems, D. phase transitions

In a disordered suspension of superconducting granules placed into an external magnetic field, the interactions produce a broad distribution of magnetic fields on the surface of the granules. As a consequence, transitions induced, for example, by increasing the external applied field or by external radiation, should occur for very different values of the applied field or the energy of the radiation than if the granules were isolated, and also different for different granules of the same sample. On the other hand, and due to the long-range nature of the diamagnetic interaction, each transition occuring in the system changes the surface field values for the whole system, and therefore these disorder effects can completely change as succesive transitions occur.

These effects are relevant in the analysis of systems like Superheated Superconducting Granule [SSG] detectors, which are being studied for the detection of neutrinos, dark matter, and other particules [1–3]. In these detectors, superconducting granules are maintained in a metastable state by an external field, and when incident radiation with enough energy scatters over a granule, the normal phase can nucleate and the transition of the granule can be detected. Interpretation of experimental results are here hampered by the spreading of the transition fields of the sample, which can be typically around 20% [4–7].

In this paper we perform numerical simulations of the succesive transitions occuring in completely disordered dispersions of superconducting spheres when the external field is slowly increased from zero. We obtain quantitative results of the surface magnetic field on the superconducting spheres during these transitions. Simulations show that the configurations obtained after several transitions are much more ordered, in the sense that the positions of the remaining superconducting granules are not completely random. On the contrary, the spheres tend to keep apart from each other and, as a consequence, the local surface fields become more uniform in the system. This has important practical consequences in SSG detection.

In previous works [8, 9], some simulations were made within an unsystematic (and not quite correct, as pointed out in Ref. [10]) scheme of dipolar and two-body approximations, in principle limited to very low densities. In this study the numerical procedure allows us both to consider the complete multi-body problem and to reach multipolar contributions of arbitrary order [11]. For the simulations, we suppose that the superconducting microgranules are microspheres of equal radius, much longer than the London lengh depth, which remain superconductor until their local magnetic surface fields reach a threshold value B_{th}. This value, which for a free of defects sphere would be equal to the superheating field B_{SH}, can vary from

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sphere to sphere due to the possible presence of surface defects. We also suppose that the transition in each sphere is complete once this threshold value is reached, so we do not consider partial transitions. The surface magnetic field is calculated by solving the Laplace equation with the appropriate boundary conditions by an expansion in multipoles [10], and an iterative method that allows us to control the convergence of the calculus to the desired precision [11]. For each configuration, N spheres are placed at random in a volume given by the desired filling factor ρ (fraction of volume occupied by the microgranules), and an external magnetic field B_{ext} is slowly increased from zero. When the maximum local magnetic field on the surface of any sphere reaches its threshold value B_{th}, it transites and the configuration becomes one of $N-1$ superconducting spheres. The process is repeated until all spheres transite. After several transitions the volume fraction occupied by the remaining superconducting spheres will be lower than the initial value ρ, and will correspond to an effective filling factor ρ_{ef}.

The numerical simulations were carried out over several configurations with a number $N = 150$ of superconducting spheres placed at random in a box of dimensions $L \times L \times 0.1L$. Values of ρ ranged from 0.01 up to 0.20. We employed a distribution of values of B_{th} experimentally determined for tin microspheres dispersed in parafine [12]. This distribution was fitted by a parabolic distribution in a range of values between $0.8B_{SH}$ and B_{SH} [10, 13].

Typical results of simulation of field-induced transitions are presented in Fig. 1. In this figure, the fraction f of still superconducting spheres (*i.e.* $f = \rho_{ef}/\rho$) versus the (increasing) external field is presented for several values of the initial filling factor ρ. The dashed line represents the expected transitions in the limit of large dilution, where the transitions would occur for each sphere when its B_{th} value equals $3/2B_{ext}$. Local fields are strongly affected by magnetostatic interactions for dense samples, and as a result transitions begin to occur for smaller applied fields.

Interactions between spheres induce changes in the surface magnetic field in each transition. In Fig. 2, the distributions of maximum surface magnetic fields for a configuration with initial $\rho = 0.20$ are presented, which after succesive transitions reach different values of ρ_{ef}. As is shown, the surface magnetic field values become more similar as the number of transitions increases, and then the width of the field distribution decreases. In Fig. 3, the mean value and the standard deviation of these distributions are presented as functions of ρ_{ef}, confirming this homogenicing effect. Note that the limiting value of B_{max} is nothing but the isolated-sphere value $3/2B_{ext}$. In the inset of the same figure higher moments (skewness and kurtosis) are shown. Values of kurtosis, being very close to zero for the dense random configuration, evolve to values very close to -1 after several transitions. Therefore the distribution of fields is much flatter than the gaussian one. On the other hand, values of skewness drop rapidly to zero with transitions, *i.e.* random configurations present a strongly assymmetrical field distribution that evolve to symmetrical for transited samples.

This behaviour is the result of the evolution of the spatial distribution of superconducting grains in a system that undergoes field-induced transitions. In Fig. 4 we compare two configurations. In the first we see the $N = 25$ spheres that remain superconducting after an increase of the external magnetic field over a system of $N = 150$ superconducting spheres placed at random with $\rho = 0.20$. In the other, we see the spatial distribution of $N = 25$ spheres placed at random in the same volume. As can be seen, the transitions have induced a certain order in the system. The first spheres that transite have particularly strong surface fields due to the presence of some other very close sphere. In this way, the succesive transitions produce a more homogeneous spatial distibution, as much in the spatial sense as in the magnetic surface field distribution. In the geometrical aspect, our results are similar to what was obtained for two-dimensional samples in Ref. [8]. However, in our simulations there is no indication of the clustering effect that was predicted in the same reference for three-dimensional samples from mean-field arguments. With regard to the distribution of maximum magnetic fields, the simulations of Ref. [9] (for $\rho = 0.10$) show a symmetrical gaussian-like distribution that evolved to a strongly assymetrical distribution after transitions. However in our results skewness and kurtosis data reveal an assymmetrical distribution of maximum field values, evolving to a non-gaussian symmetrical distribution as transitions occur. We can conclude that the correct calculation of higher order multipolar and multibody contributions to the magnetic field is an essential ingredient of these kind of simulations.

Following the idea of an ordering process induced by transitions, it is interesting to compare our results with what obtained in previous studies, involving systems without any transitions [11]. Results reported in Ref. [11] showed that the surface field statistics is very sensitive to the configurational order present in the system, and in particular disordered samples present surface field statistics very different from that of regular systems for any value of the concentration. In the case of perfectly regular (square periodic) arrays

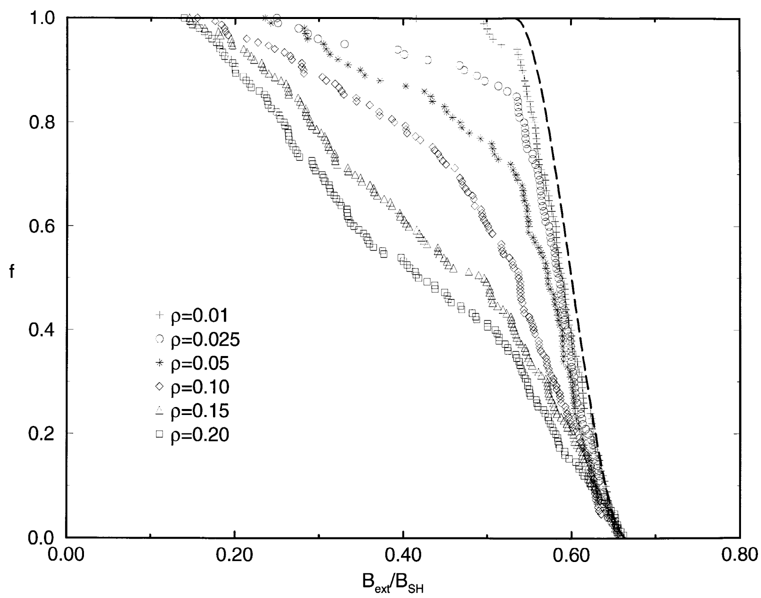

Fig. 1. Fraction f of still superconducting spheres vs B_{ext}/B_{SH}, after an increase of the external magnetic field from zero, for several samples of $N = 150$ initially superconducting spheres, corresponding to different initial concentrations ρ. Discontinuous line corresponds to the dilute limit, i.e. assuming a maximum surface field of $3/2B_{ext}$ for all the spheres.

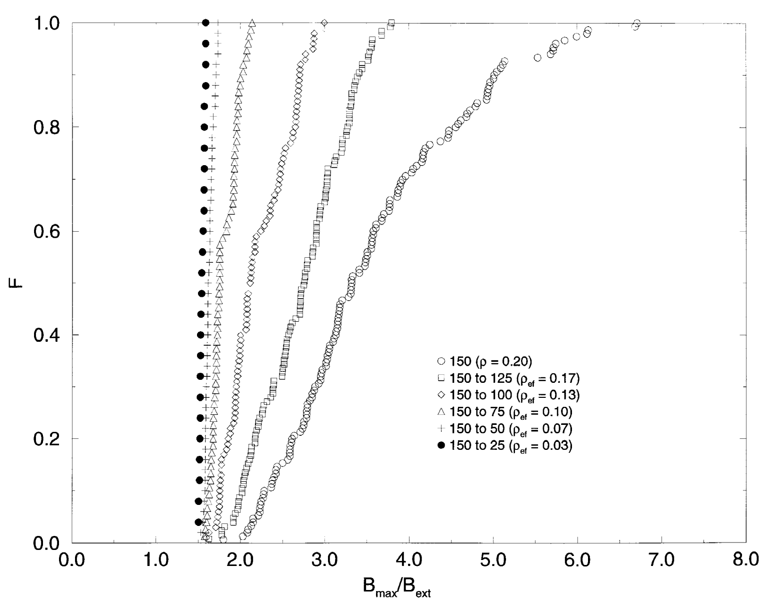

Fig. 2. Fraction F of spheres with maximum surface field lower than the x-axes value (in units of B_{ext}), in the evolution of a configuration with initially $\rho = 0.20$ and $N = 150$ to several final values of N and ρ_{ef}.

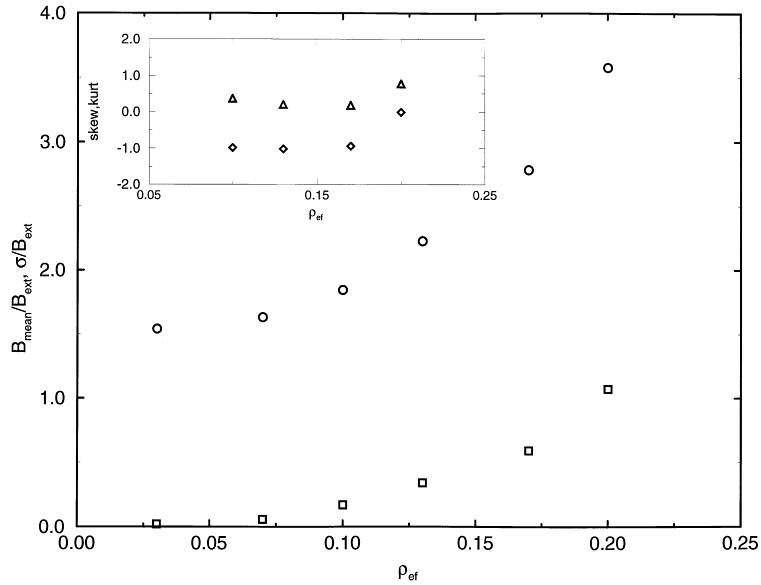

Fig. 3. Mean value (circles) and standard deviation (squares) of the distribution of maximum surface field for evolution of a configuration with initially $\rho = 0.20$ and $N = 150$ as a function of ρ_{ef}. Results are averages of 5 independent samples. In the inset the corresponding skewness (triangles) and kurtosis (diamonds) are shown for $\rho_{ef} \geq 0.10$ (statistical errors for these quantities are too large for smaller ρ_{ef}, owing to the small number of superconducting spheres involved).

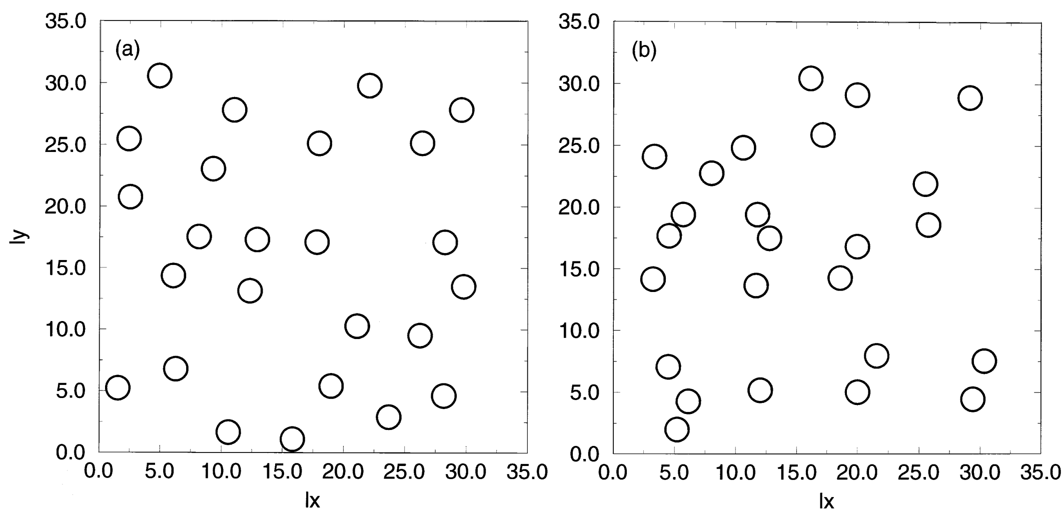

Fig. 4. Spatial distribution for $N = 25$ spheres: (a) remaining superconducting after transitions from $N = 150$, (b) placed at random.

of superconducting granules skewness and kurtosis presented large values in the most concentrated samples, which decreased in the limit $\rho \to 0$ [11]. On the other hand, the corresponding results for completely random samples showed the opposite behaviour, with gaussian-like distributions for the denser samples (with small skewness and kurtosis), but with strongly assymmetrical and non-gaussian distributions for small densities (with very large values of skewness and kurtosis). In our case, dilute systems resulting of field-induced transitions of concentrated samples are distinctly different from random systems with the same concentration.

We have also compared the behaviour of several configurations with different initial ρ when, after several transitions, they reach the same final ρ_{ef}. Previous simulations showed [11] that, for the same external magnetic field, random configurations had surface field distributions whose width increased strongly with ρ. In the present simulations, when these systems

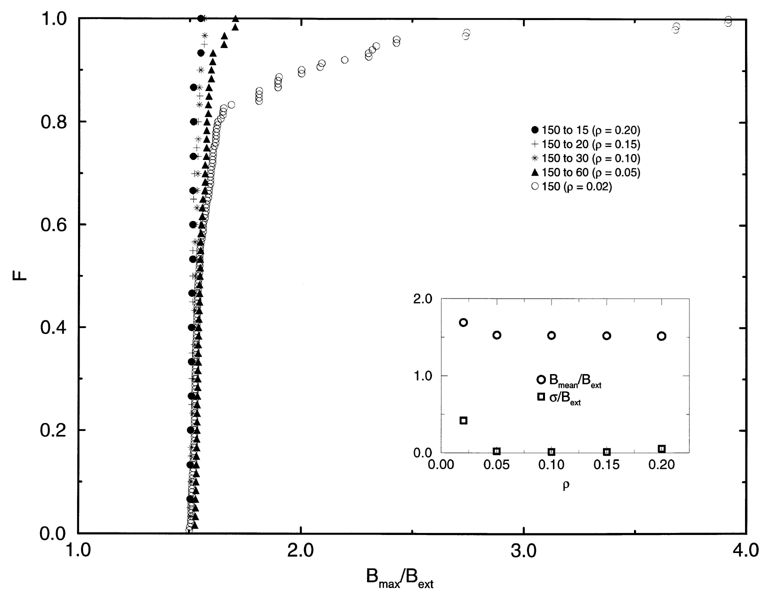

Fig. 5. Fraction F of spheres with maximum surface field value lower than the x-axes value (in units of B_{ext}) for configurations with different initial ρ and the same final $\rho_{ef} = 0.02$. In the inset the corresponding mean value (circles) and standard deviation (squares) as a function of ρ.

evolve towards the same ρ_{ef}, the order induced by the succesive transitions produces a final situation with much smaller field spreading.

This is shown in Fig. 5, where one may see the maximum surface field distribution for configurations with different initial values of ρ that have undergone transitions until all of them have reached the same final $\rho_{ef} = 0.02$. They are compared to a configuration at random corresponding to an initial $\rho = 0.02$. As can be seen in this figure, the field values for still superconducting spheres occur within a narrow interval for the systems that have already suffered transitions. This effect is stronger in configurations coming from higher initial ρ; that is, those having undergone a higher number of transitions. On the other hand, in the random configuration with $\rho = 0.02$, despite its low density, the disorder effects over the field distribution are evident. In the inset of the same figure, the mean value and standard deviation of the corresponding distributions are represented as a function of the initial ρ. The constancy of the mean value, which approaches the isolated-sphere value $\frac{3}{2}B_{ext}$, and the vanishing value of the standard deviation, confirm the homogeneity of the transited systems compared with the random one. Similar behaviour has been obtained for higher values of ρ_{ef}: whereas a random dense system is characterized by a very wide field distribution [11], after several transitions the width is drastically reduced.

In these results, the tendency to regularity or homogeneity is latent as a consequence of the order induced by the succesive transitions. It is posible to measure this order by noting the distance between still superconductor spheres. In Fig. 6, the mean value of the distances between each sphere to the nearest one is shown, corresponding to the same configurations as in Fig. 5, as a function of their initial ρ. In this figure, all the configurations have the same effective filling factor, but the distances between superconducting spheres increase strongly with the initial ρ, and consequently with the number of transitions.

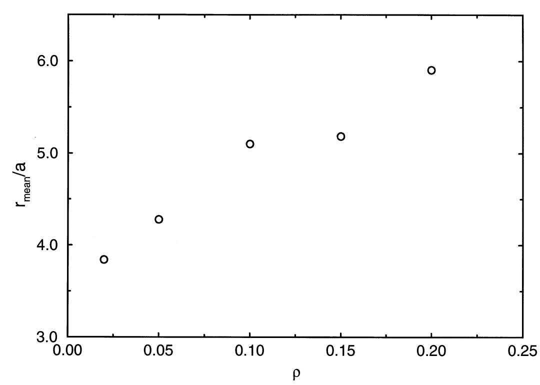

Fig. 6. Mean value of the distances between each sphere to its nearest one, in units of the radius of the spheres a, for systems evolved to the same final $\rho_{ef} = 0.02$, versus the initial ρ.

We conclude that the succesive transitions induced by an external magnetic field provide a strong ordering mechanism in disordered SSG configurations. Moreover statistical properties of surface field distributions critically depend on the configurational order of the system. Spatial positions and surface field distributions are more homogeneous (for a fixed concentration of superconducting spheres) when the sample is the result of several transitions in a system with higher initial density, rather than in a sample with spheres placed directly at random. As a consequence, the energy threshold for transitions in SSG detectors has an uncertainty that can be reduced by the ordering induced by the external field. Numerical simulation of these systems, by solving the complete Laplace equation for the magnetic field, appears as a promising tool in the analysis of SSG experiments.

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