Intial Fluctuation in Velocity Field and Mode Variation of Taylor-Couette Flow

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ABSTRACT

Taylor-Couette flow starting from rest with a variation of an acceleration rate of a cylinder rotation shows various flow modes even at a constant final Reynolds number and a fixed aspect ratio, and it also exhibits some probabilistic characteristics that may be due to generic nature of chaos, imperfections of experimental apparatus and dispersions of conditions. An important disturbance is velocity fluctuations in an initial fluid state. In this study, the relationship between fluctuations in an initial state and modes of flow patterns are considered. The effect of quasi-isentropic initial flows with various intensities on the final modes of flows that start from rest and accelerate gradually is investigated numerically.

INTRODUCTION

Studies on the transition of phase are an important subject to understand nonlinear dynamical systems, and one of the typical phenomena is Taylor-Couette flow developing between two cylinders with finite length. The flows starting from rest were investigated experimentally and numerically [1-3]. It is well known that this flow has a multiple mode even at fixed Reynolds number, aspect ratio and radius ratio [4]. Beilek and Koschmieder [5] observed the development of nonunique flows. The numerical study by Furukawa et al. [6] showed that the final flow modes depend on the acceleration rate of the inner cylinder.

It is almost impossible to obtain perfect conditions of experiments. Even though experiments were carried out under one's careful attention, series of trials to identify final modes presented statistical variations of observed results [7]. It was also suggested that the asymmetry of experimental apparatus made the flow collapse toward a preferred state among branching solutions [8, 9]. Some of imperfections other than asymmetry of apparatus are variations of the acceleration rate of the cylinder and viscous scales of velocity components in initial conditions.

In this paper, the velocity fluctuation in viscous length scale of an initial state is considered. Quasi-isotropic flows with various velocity intensities are introduced as initial states, and the flow modes appearing from these initial states are investigated.

NUMERICAL METHOD

The gap between cylinders is indicated by D and the length of cylinders is H, so the aspect ratio Γ is given by H/D. The radius ratio of two concentric cylinders is η. The values of η and Γ are fixed at 0.667 and 4.0, respectively. The Reynolds number is defined by using the characteristic length that is the gap between cylinders and the characteristic velocity that is the maximum circumferential rotation velocity attained during each calculation. All physical parameters are made dimensionless by using the characteristic length and the characteristic velocity.

The governing equations are the unsteady axisymmetric Navier-Stokes equations and the equation of continuity, which are expressed in the cylindrical coordinates (r, θ, z) with velocity components u, v, w. Suitability of axisymmetric equations of this problem is considered in [6]. The inner cylinder rotates and the outer cylinder and end walls are stationary. We assumed that the lower end wall is at z = 0. The angular velocity of the rotating inner cylinder is accelerated linearly from rest to the final velocity ω_0. The acceleration parameter St is given by $St = 1/\omega_0 T$. The basic solution method is the MAC method. The boundary condition of the velocity is the non-slip condition.

QUASI-ISOTROPIC VELOCITY FIELD

The experimental result by Nakamura and Toya [7] is compared with our numerical result. Their experiment showed that the local velocity components in a quiescent state had values ranging from 0.14 mm/s to 0.28 mm/s. Snyder [10] proposed that the diffusion time across the gap was D^2/ν and the velocity in the viscous scale was ν/D, where ν is the kinematic viscosity of fluid. In the experiment [7], the velocity scale suggested by Snyder is estimated at 0.35 mm/s, which shows a favorable agreement with the experimental observation.

At first, random values generated by the L'Ecuyer's method [11] are assigned to values of velocity components u, v, w. Then unsteady numerical calculation is carried out by using these random values and a flow field that satisfies the equation of continuity is obtained. Repeating this procedure N times, we get an ensamlbe average of flow fields in order to reduce the dependences of initial radomnesses. The ensemble average and the deviation of the velocity vector, $\bar{\boldsymbol{u}}$ and $\hat{\boldsymbol{u}}$, are given by

$$\bar{\boldsymbol{u}} = \frac{1}{N}\sum_{1}^{N} \boldsymbol{u}\,, \qquad \hat{\boldsymbol{u}} = \sqrt{\frac{1}{N}\sum_{1}^{N}(\boldsymbol{u} - \bar{\boldsymbol{u}})^2}\ . \tag{1}$$

The proportionality relation given by the formula $\hat{\boldsymbol{u}}^2 \propto t^{-1}$ is used to conclude that the ensemble-averaged flow field is isotropic [12], where $\hat{\boldsymbol{u}}$ is obtained at the midpoint in the r-z section and t is the dimensionless time. We confirmed that the spatial average of $\bar{\boldsymbol{u}}$ was much smaller than that of $|\bar{\boldsymbol{u}}|$. The ensemble-averaged flow field is normalized to make the velocity intensity have the value observed by experiments. Initial conditions, 105 in total, are obtained by multiplying the intensity of the normalized flow field by values from 0.01 to 5.0.

NUMERCAL RESULTS

Figure 1 shows frequency distributions of final flow modes that develop from 105 initial states generated by the method mentioned in the previous section. The Reynolds numbers and the acceleration parameters are the same as those used in the experiment by Nakamura and Toya [7]. Figures 1 (a), (b), (c), (d) and (e) are the frequency distributions at Re = 500, 750, 1000, 1250 and 1500, respectively, and (f) is the frequency distributions at Re = 1500 in experiment [7]. A mode notation Nn stands for a normal mode with n cells, and An represents an anomalous mode with n cells. The frequency distributions did not show clear dependence on the intensities of initial velocity fields. The result at Re = 250, which is not shown in this paper, showed that only normal four-cell mode appears at every acceleration parameter. At Re = 500, the anomalous five-cell mode is found and the value of its frequency becomes large as the acceleration parameter increases. The normal four-cell mode is dominant at Re = 750. At higher Reynolds numbers of 1000, 1250 and 1500, various final modes establish. The primary mode at this aspect ratio Γ = 4.0 is the four-cell mode, and this mode appears when acceleration parameter is small and the acceleration time of the inner cylinder is long.

The complexity of the frequency distribution is represented by entropy function,

St	0.09	0.18	0.27	0.36	0.45	0.54
N2	1	0	0	0	0	0
A3	0	0	0	0	0	0
A4	0	1	1	1	1	1
N4	104	104	76	45	26	6
A5	0	0	28	59	78	98

(a) Re = 500

St	0.04	0.08	0.12	0.16	0.19	0.23
N2	0	0	0	0	1	0
A3	0	0	0	2	1	0
A4	0	0	0	0	0	1
N4	105	99	100	100	103	104
A5	0	6	5	3	0	0

(b) Re = 750

St	0.02	0.04	0.06	0.09	0.11	0.13
N2	0	0	0	0	0	0
A3	0	0	19	1	0	3
A4	0	0	1	88	48	40
N4	104	104	11	6	3	5
A5	0	0	74	10	54	57

(c) Re = 1000

St	0.01	0.03	0.04	0.06	0.07	0.09
N2	1	0	0	0	0	0
A3	0	0	14	3	17	1
A4	0	1	1	25	52	27
N4	104	104	13	7	7	31
A5	0	0	77	70	29	46

(d) Re = 1250

St	0.01	0.02	0.03	0.04	0.05	0.06
N2	0	0	0	0	0	0
A3	0	0	0	25	3	4
A4	0	0	16	3	45	17
N4	102	102	32	48	45	57
A5	3	3	57	29	12	27

(e) Re = 1500

St	0.01	0.02	0.03	0.04	0.05	0.06
N2	1	4	5	3	2	4
A3	3	25	29	22	15	22
A4	0	0	7	1	1	4
N4	96	71	59	74	82	70
A5	0	0	0	0	0	0

(f) Re = 1500(Experiment [7])

Fig. 1 Frequency distribution of final modes.

$$H = \sum_i \left(-p_i \log p_i\right), \qquad (2)$$

where p_i is the probability of an outcome of a final flow mode at a fixed Reynolds number and a fixed acceleration parameter. Figures 2 (a), (b), (c), (d) and (e) show variations of the entropy function against the acceleration parameter. The profiles at *Re* = 500 shows its peak and the profile at *Re* = 750 has very low value. At higher Reynolds numbers, the value of the entropy function tends to be large as the acceleration parameter increases.

Series of mode transitions leading to a final mode may not unique and they depend of the intensity of the initial fluctuation. As an example, the normal four-cell mode developing from two transitions found at *Re* = 500 and *St* = 0.0894 are shown here. In the first transitions, vortices appear on end walls of cylinders, and many cells emerge, and a merging and disappearing of vortices make a final state of the normal four-cell mode. In the first transition, first vortices appear on end walls of cylinders, and many cells emerge and an intermediate state of the normal six-cell mode appears. This mode remains stable for a while, and the final four-cell mode is formed by weakening a pair of vortices.

Some of the causes of multiple flow modes are considered to be an imperfection of experimental apparatuses and a dynamical drift in a quiescent condition. The imperfections of apparatuses may be due to an asymmetry and eccentricity of cylinders and variances of a rotation rate of a driving motor. Another cause of imperfections is the fluctuation of velocity components in the initial state. In addition, the effect of the irregularity of the rotation speed of the inner cylinder could be important. We assumed the variations of the acceleration parameter with 10 %. Though the details of the results are not shown here, it has been concluded that the final flow modes are almost independent from the variation of the motor acceleration considered here.

Bestehorn and Haken [13] studied the dynamical disturbances and estimated numerically the effect of incompleteness in the initial condition on the Bénard instability. They reported that different initial velocity fields with very weak roll patterns disturbed randomly made the fluid remember final states of Bénard convection patterns with different symmetry, and that the procedure completing symmetric structures was considered to act as an associative memory used in pattern recognitions. In the present study, initial states given by a quasi-isotropic state with various intensities have been used and it is confirmed that a slight change of the intensity brings different final modes and different processes of mode formations.

The final flow modes obtained by the experiment [7] and the present numerical analysis are summarized in Table 1. The agreement between experimental and numerical results is reasonable. Some differences are as follows. The experimental result shows the anomalous five-cell mode at *Re* = 500 and 750, and the numerical result predicts the mode at every Reynolds number except 250. The only case where the normal six-cell mode appears is the experiment at *Re* = 500.

The present numerical analysis predicted the normal six-cell mode as an intermediate state of a mode transition leading to another flow mode. In order to obtain the normal six-cell mode in experiment, the rotation rate of the cylinder was carefully adjusted to

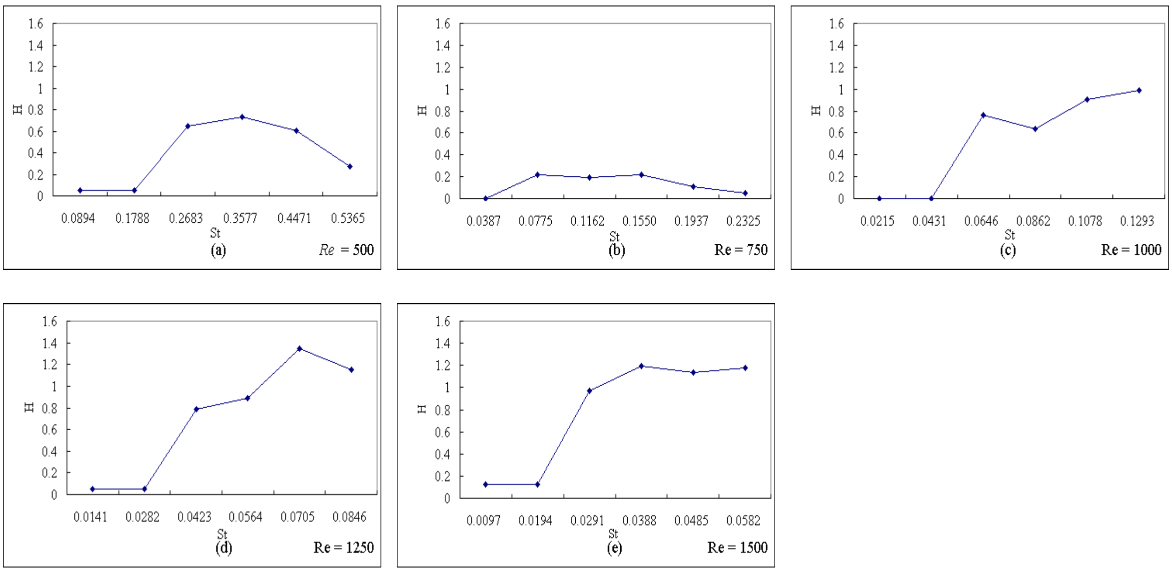

Fig. 2 Variation of the entropy function against acceleration parameter.

Table 1 Comparison of final flow mode.

	Experiment	Numerical Calculation		Experiment	Numerical Calculation
Re = 250	N4	N4	*Re* = 1000	N2, A3, N4, A4	A3, N4, A4, A5
Re = 500	N2, A3, N4. A4, A5, N6	N2, A3, N4, A4, A5	*Re* = 1250	A3, N4, A4	N2, A3, N4, A4, A5
Re = 750	N2, A3, N4, A4, A5	N2, A3, N4, A4, A5	*Re* = 1500	N2, A3, N4, A4	A3, N4, A4, A5

stabilize the flow before the vortices of the normal six-cell mode once formed were collapsed [7]. Numerical and experimental results have shown that the only final flow mode at *Re* = 250 is the normal four-cell mode. The normal four-cell mode is frequently formed at *Re* = 750 and several anomalous modes appear at relatively high Reynolds numbers of *Re* = 1000, 1250 and 1500.

Initial conditions of 105 cases are assumed to occur with uniform probability, though it cannot be expected in experiment. The problem on the statistical correction to fit the numerical simulation to experimental conditions will be considered in future.

SUMMARY

The effect of quasi-isotropic random initial flows with various intensities on flows developing in Taylor-Couette system has been investigated numerically. The inner cylinder is accelerated from rest at several acceleration rates. The complexity of the frequency distributions of the final modes is represented by the entropy function. The result of the final flow modes obtained by the numerical study well predicts the modes observed by experiments. Various modes appear at higher Reynolds numbers and larger acceleration parameter, though no distinct relations among the final mode, Reynolds number, acceleration parameter and intensity of inital velocity are found. Multiple transition processes leading to a unique final mode have been found.

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