Baroclinic instabilities in a thermally driven rotating annulus

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ABSTRACT

1 Motivation

Baroclinic instabilities are important for the transport of heat and momentum in the atmosphere and oceans of the earth. Systematic investigations of this phenomenon have been carried out since the middle of the last century.

Until now, it is not possible to analyse the complex processes of baroclinic waves completely, neither in theory nor in laboratory experiments. Numerical approaches are restricted because of the non–linearity of the underlying equations. Therefore, laboratory experiments are necessary for the investigation of the basic dynamic processes.

We study experiments of baroclinic instabilities in a different heated rotating annulus. The methods of non–linear time series analysis enables us to investigate the dynamics of baroclinic waves, particulary in transition zones between different flow regimes. Characteristic time series can be recorded with the optical Laser Doppler Velocimetry (LDV) technique. Flow visualisation technique is also used.

2 Experimental setup

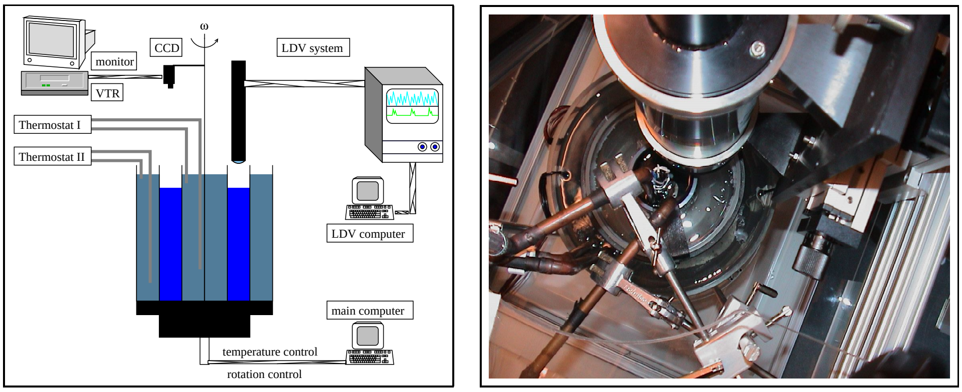
The rotating annulus (figure 1) has three concentric chambers. The inner and outer chamber is filled with de-ionised water and connected with two thermostats. They control the wall temperatures of the cylindrical gap between the chambers which is measured by thermocouples. All surfaces are free. The inner wall is made of aluminium, the outer walls are made of plexiglass. The whole tank is mounted on a turntable. The main computer controls the rotation rate and the temperature data. The LDV system has an additional control computer. Pictures of the surface flow taken with a co–rotating CCD camera which is attached to the rotating axis are stored on videotape (VTR). A Dantec–LDV system is used to measure long velocity time series of at most 9 hours. The aspect ratio of the system is 4.4, de-ionised water with a kinematic viscosity of 1.004 mm^2/s is used as the fluid.

3 Experiments

Figure 2 shows regions of baroclinic waves in the parameter space for water (Hide and Mason [1970], modified). Different flow regimes were observed depending on the rotation rate and temperature gradient respectively (e.g. Früh and Read [1997]). The dimensionless Taylor number Ta and thermal Rossby number Ro is defined as follows:

$$Ta = \frac{4 \cdot \Omega^2 \cdot (b-a)^5}{\nu^2 \cdot d} \qquad Ro = \frac{g \cdot d \cdot \Delta\rho}{\overline{\rho} \cdot \Omega^2 \cdot (b-a)^2} \tag{1}$$

where d = depth of the fluid, Ω = angular velocity, a,b = inner and outer radius of the gap, ρ = density of the fluid, ν = kinematic viscosity, g = gravity constant.



(a) schematic diagram **(b)** top view image

Figure 1: experimental setup.

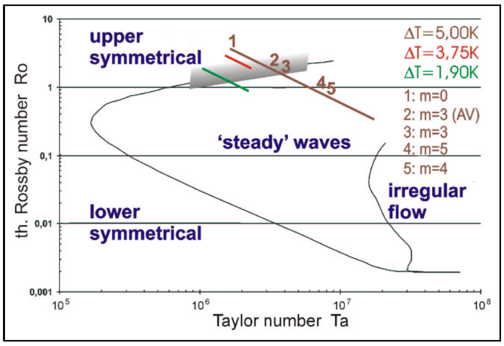


Figure 2: regions of baroclinic waves in the parameter space for water (Hide and Mason [1970], modified).

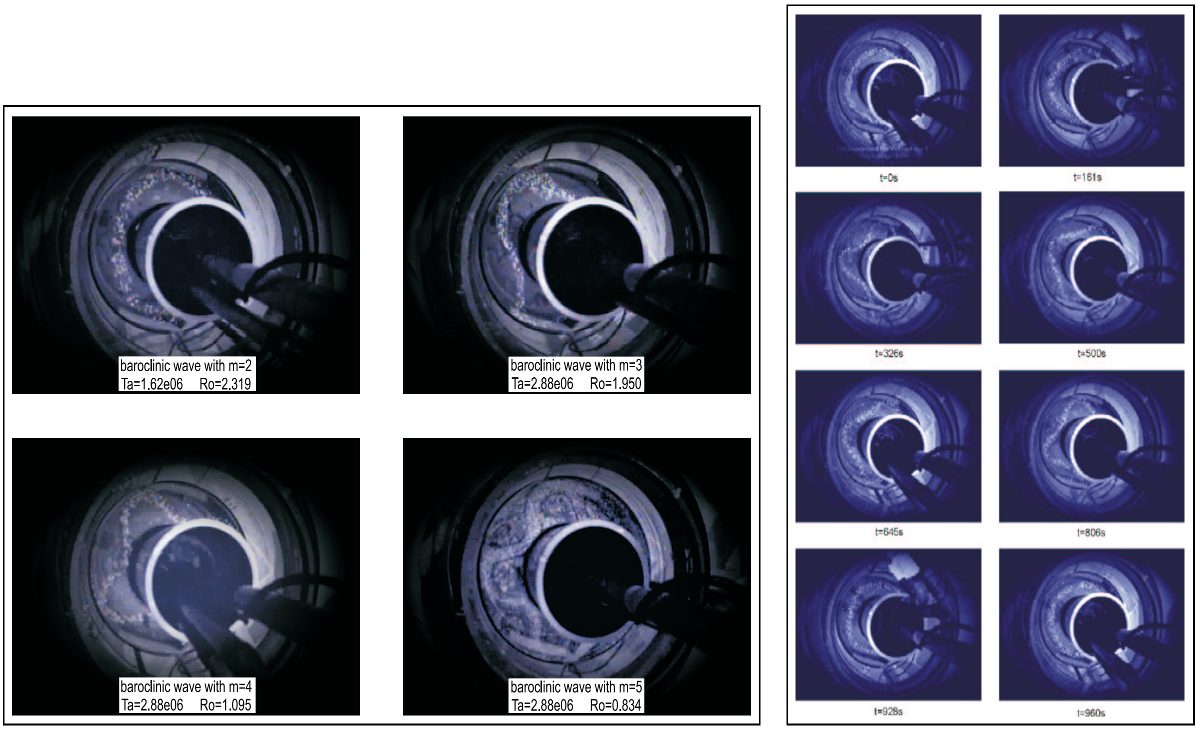
We investigate flow regimes, particularly in the first transition zone from axisymmetric basic flow to stable baroclinic waves (shaded), where low dimensional chaotic flow as well as stable flow occurs (Sitte and Egbers [2000]). The coloured lines indicate points of LDV measurements. The radial velocity component of the flow is measured approx. 20 mm below the fluid surface.

4 Flow visualisation

Pictures of the surface flow which is visualised with aluminium flakes can be seen in figure 3. Figure 3(a) shows the surface flow of steady baroclinic waves with different wave number m. Waves with m=2 to m=5 are identifiable. Figure 3(b) shows a baroclinic wave (m=4) with an oscillating amplitude recorded at different times t (Ro=1.73, Ta=$2.10 \cdot 10^6$). The cycle duration of the oscillation is about 1000 s. Such 'amplitude vacillation' (AV) waves occur in the first transition zone and were also observed by other research groups (e.g. Read et al. [1992], Früh and Read [1997], Sitte and Egbers [2000]). Also waves with a 'modulated amplitude vacillation' (MAV) were observed which oscillation frequency is not constant in contrast to AV–waves.

5 LDV measurements

To detect the dynamic behaviour of baroclinic waves the time series is analysed with linear and non–linear methods. The method of time delayed coordinates by Takens [1980] is used to reconstruct the attractor of the system. Then, dynamic variables can be calculated to characterise



(a) stable baroclinic waves with m=2 to m=5

(b) amplitude vacillation wave (m=4, AV)

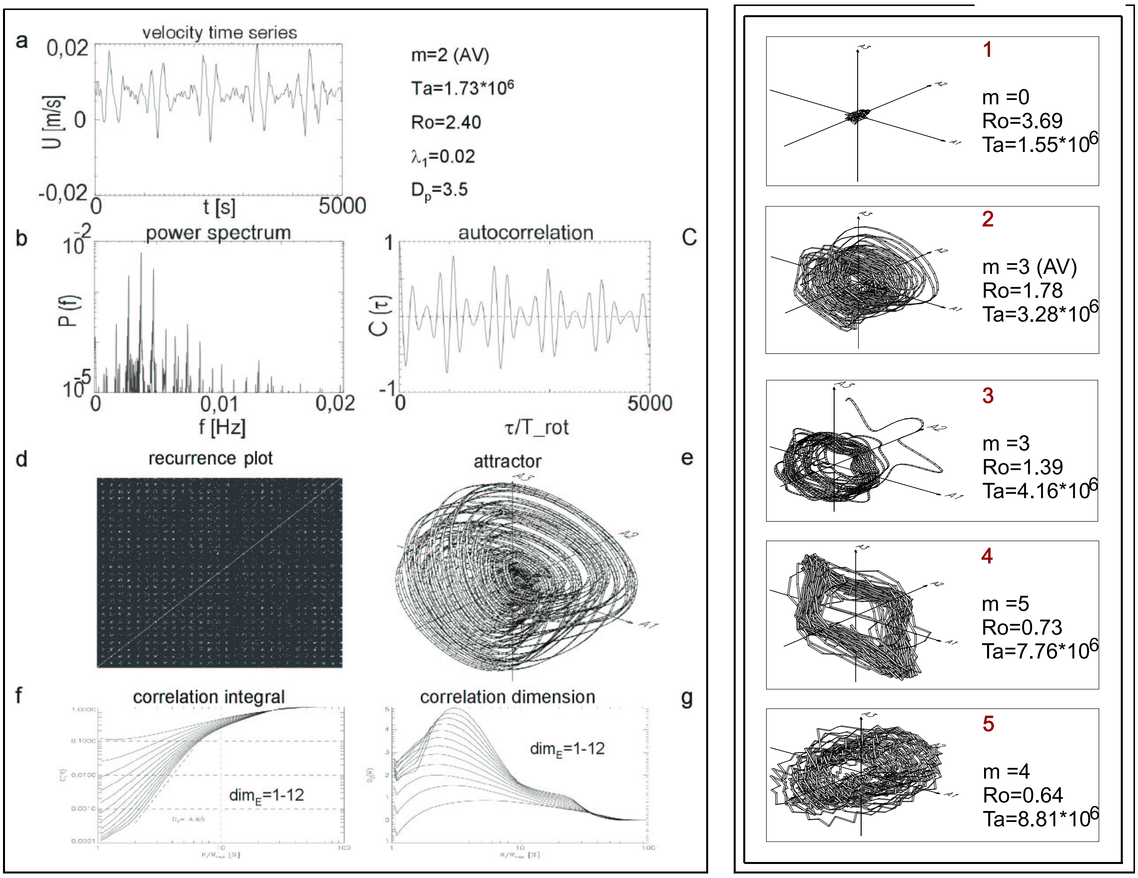
Figure 3: pictures of the surface flow of baroclinic waves.

the complexity of the flow. The geometric structure of an attractor can be analysed with fractal dimensions. Here, the correlation dimension D_2 and the pointwise dimension D_p are calculated. Recurrence plots are also constructed.

An analysis of a baroclinic wave (m=2, AV) can be seen in figure 4(a). Here, picture (a) shows a fraction of the time series. The drift frequency (f_c=0.0037 Hz) is surrounded by several peaks at a constant distance of 0.001 Hz (see (b)). The autocorrelation function (c) shows a very slow decay. The attractor (e) is characterised by the overall settled character of the time series. The recurrence plot (d) has no axially parallel lines which indicates chaotic dynamic flow. The largest Lyapunov exponent is significant greater than zero (λ_1=0.02). The pointwise dimension (D_p=3.5) is of the order of D_2 (f, g). The calculated values indicate low dimensional chaotic flow at this parameter point.

Figure 4(b) shows different attractors which are calculated from time series at parameter points no. 1 to 5 in figure 2. From picture 1 to 5 the angular velocity increases but the horizontal temperature gradient remains constant. A pointwise attractor (1) follows from an axisymmetric basic flow, where the radial velocity component is almost zero. Picture 2 shows the attractor of an oscillating wave (m=3). Attractors of steady baroclinic waves with m=3 and m=5 can be seen in picture 3 and 4 respectively. Here, it is a torus in contrast to the more complex run in picture 2. An attractor of a steady baroclinic wave (m=4) with a more variable amplitude shows picture 5.

Recapitulating, flow regimes with a totally different dynamic state exists close–by. Furthermore, hysteresis is found but not discussed here.



(a) non–linear analysis of an AV–wave (m=2)

(b) attractors of different flow regimes

Figure 4: time series analysis of LDV measurements.

REFERENCES

[1] Früh, W.–G. and Read, P.L. [1997] *Wave interactions and the transition to chaos of baroclinic waves in a thermally driven rotating annulus*, Phil. Trans. Roy. Soc. Lond. A, **355**,101-153

[2] Hide, R. and Mason, P.J. [1970] *Baroclinic waves in a rotating fluid subject to internal heating*, Phil. Trans. Roy. Soc. Lond. A, **268**,201-232

[3] Read, P.L., Bell, M.J., Johnson, D.W. and Small, R.M. [1992] *Quasi-periodic and chaotic flow regimes in a thermally driven, rotating fluid annulus*, J. Fluid Mech., **238**,599-632

[4] Sitte, B. and Egbers, C. [2000] *Higher order dynamics of baroclinic waves*, Physics of Rotating Fluids, Proc. 11th Int. Couette–Taylor Workshop, Lecture Notes in Physics, Springer, **549**, 355-378

[5] Takens, F. [1980] *Detecting strange attractors in turbulence*, Dynamical Systems and Turbulence (ed. D. Rand and L.S. Young), Lecture Notes in Mathematics, Springer, **898**, 366-382