

Influence of Rim-Shroud Clearance on Flow around Rotating Disc in Cylindrical Enclosure

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ABSTRACT

Three dimensional numerical study has been conducted to simulate the flow around a disc rotating in a stationary cylindrical enclosure, and the growth of disturbances and the effect of the clearance between the disc rim and the shroud have been investigated. A quantitative method to estimate the axisymmetry of flows is introduced and the critical Reynolds number at which an asymmetric spiral flow appears is evaluated. The clearance between the rim and the shroud largely influences the entire flow structure on the rotating disc and the stationary walls of the enclosure, and it promotes the transition to spiral rolls and turbulent spiral rolls.

INTRODUCTION

Flows induced by rotating objects appear in turbomachinery, storage devices of computers and ocean circulations, and the investigation of these flows is very interesting. One of the typical cases of these phenomena is the flow which develops between a rotating disc and a stationary disc, and the flow has been investigated numerically and experimentally [1-3]. In these studies, it has been shown that, as the Reynolds number based on the rotation rate of a disc increases, circular rolls, spiral rolls and turbulent spiral rolls (or solitary waves) appear, and then the flow becomes turbulent. However, the disc is assumed to be very thin rigid wall that produces boundary conditions of velocity components, and the geometrical shape of the disc is not considered.

Between the rotating disc and the stationary disc, the Bödewadt layer and the Ekman layer develop on the stationary lid and on the rotating disc, respectively. In an ideal case where the gap width between the rotating disc and the stationary disc is infinite, instabilities arising from the presence of an inflection point in the velocity profile and the Coriolis force appear. Experiments carried out by Savas et al. [4] confirmed the appearance of spirals induced by these instabilities, together with circular waves. Recently many researches appeared, for example, see [5, 6].

In this study, we are concerned with flows induced by a rotating disc in a cylindrical enclosure. The disc is driven by a thin shaft and the radius of the disc is smaller than the inner radius of the enclosure. When we see practical devices, we need to examine the finite thickness of a rotating disc that would built up Taylor-Couette system between the rim of the disc and the shroud. The effect of the rim-shroud clearance on the flow structures is investigated.

FORMULATION

The geometrical configuration of the system is shown in Fig.1. The cylindrical enclosure with the inner radius r_c is stationary and the disc with radius r_d is rotating with the angular velocity ω . Physical quantities are made in dimensionless forms by the reference length r_c and the reference velocity $r_d\omega$. For comparison with experimental results, some parameters are fixed. The thickness of the disc h_d is 0.07, and $h_u = h_l = 0.07$, where h_u and h_l are the upper and lower gap widths between the disc and upper and lower end walls, respectively. The disc radius r_d is 0.954, 0.894 and 0.795, and the shaft radius r_s is 0.07.

The governing equations are the three-dimensional Navier-Stokes equations and the equation of continuity in the cylindrical coordinate (r, θ, z) :

$$\partial \mathbf{u} / \partial t + \nabla \cdot (\mathbf{u} \mathbf{u}) = -\nabla p + 1/Re \nabla^2 \mathbf{u} \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0 \quad (2)$$

where \mathbf{u} is the velocity vector with its components (u, v, w) , p is pressure and Re is the Reynolds number based on the reference length and velocity. Initially, the fluid is at rest in the whole system. The disc and supporting shaft suddenly begin to rotate at a given angular velocity ω .

QUANTITATIVE ESTIMATION OF AXISYMMETRIC FLOWS AND ASYMMETRIC FLOWS

As the Reynolds number increases, the axisymmetric flow disappears and the asymmetric flow appears near the shroud of the stationary enclosure. When the flow loses a uniform axisymmetry, the asymmetric flow including spiral rolls, turbulent spiral rolls and others are observed. In experiments, the rough sketch of the flow exchanges has been observed. Though visual inspections are usually used in these cases, the visual method is somewhat subjective and it is difficult to determine the criterion of the critical phenomena. In the present numerical study, we introduce a quantitative and objective method to evaluate the axisymmetry of flows by using $V(t, r)$ which is the spatial variance of the axial velocity component:

$$V(t, r) = \frac{1}{H} \int_0^H \frac{1}{2\pi r} \int_0^{2\pi} \sqrt{\left(w - \frac{1}{2\pi r} \int_0^{2\pi} w r d\theta \right)^2} r d\theta dz \quad (3)$$

After a number of trials, an axial velocity component was selected as an indicator of asymmetry. In the followings, the variance $V(t, r)$ is evaluated at $r = 0.421$ where unsteady flow patterns are more observable. Figure 2 shows the spatial variations in the θ - z plane for the axisymmetric flow and the asymmetric flow. The abscissa is θ and the ordinate is z . In Fig. 2, no variation in the circumferential direction is observed and the value of $V(t, r)$ at $t = 100.0$ is 4.39×10^{-7} . In the asymmetric flow shown in Fig. 2 (b), spatial fluctuations appear and the value of $V(t, r)$ is 1.31×10^{-3} .

NUMERICAL RESULTS

Iso-lines of the axial velocity component in the Bödewadt layer on the stationary lower lid are shown in Fig. 3, and those in the Ekman layer on the rotating disc are shown in Fig. 4. The Reynolds numbers are 39860, 41593 and 60650, the disc radius is 0.954, and the disc rotation is counterclockwise. At $Re = 39860$, the flow is a basic flow and no spiral or spot is found in the whole field except the region neat the shroud. Spiral rolls appear at $Re = 41593$ and turbulent spiral rolls emerge at $Re = 60656$.

The variations of the enstrophy, defined by the half of the squared azimuthal vorticity, in the r - z plane at $r_d = 0.954$ and 0.894 are shown in Fig. 5 (a) and (b), respectively. In both cases, the

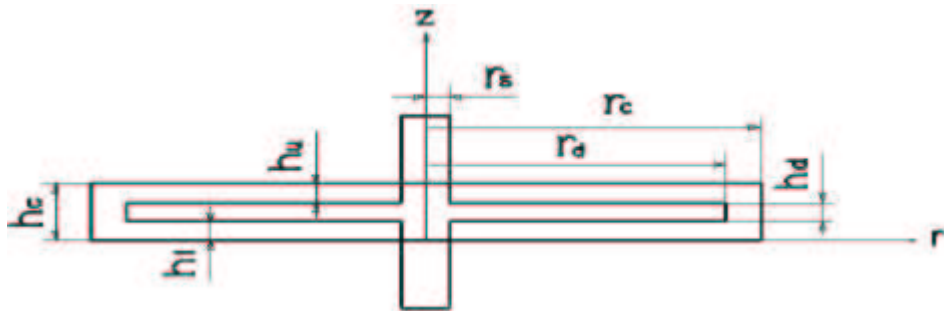


Fig.1 Geometrical configuration of flow system.



Fig.2 Variations of the axial velocity component in the θ - z plane ($r_d = 0.954$, $r = 0.412$).

Reynolds number is 39860. At $r_d = 0.954$, the flow is the basic flow, as has been seen in Figs. 3 and 4. At $r_d = 0.894$, the flow has turbulent spiral rolls. Disturbances incited by the spirals clearly appear on the stationary lids, and the two vortices of Taylor-Couette flow are unsteady and asymmetric with respect to the axial direction.

Variations of the variance $V(t,r)$ against the Reynolds number are shown in Fig. 6. When the disc radius r_d is 0.954, the value remains small by $Re = 39860$, and it increases at $Re \geq 41593$. In our experiment, it was observed that the flow at $Re = 35645$ was axisymmetric and the flow at $Re = 43327$ was asymmetric. This evidence supports that the growth of the variance is a reasonable criterion for the transition of the axisymmetric and asymmetric flows, which occurs around $Re = 41000$. When the disc radius r_d is 0.894 and 0.789, the variance begins to increase at $Re > 10000$, while the growth rate is not so high than that at $r_d = 0.954$. During the transition from the basic flow at these disc radii, not spiral rolls but weak turbulent spiral rolls appeared and they were enforced as the Reynolds number increased. At $Re < 10000$, the flow is almost a basic flow at every disc radius, and the variance remains less than 1.0×10^{-5} , which can be used as an critical value for the axisymmetry.

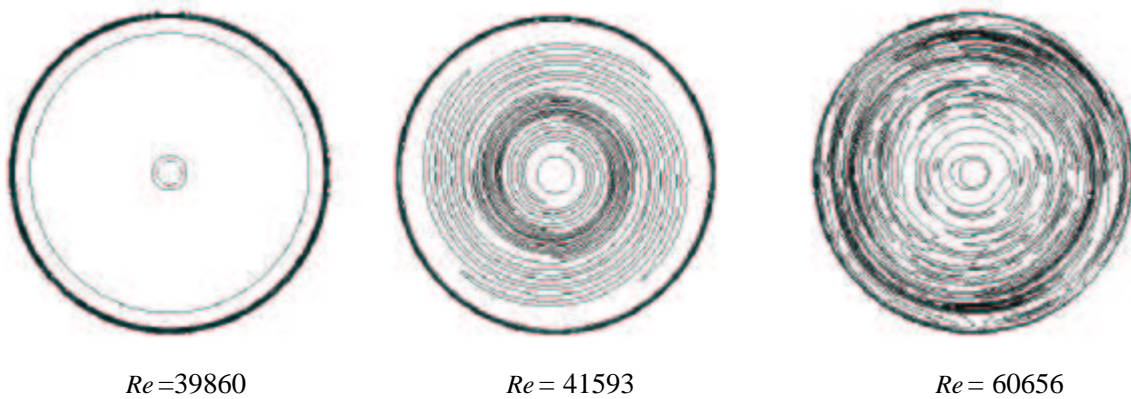


Fig.3 Iso-lines of the axial velocity component in the Bödewadt layer ($r_d = 0.954$, $z = 0.009$).

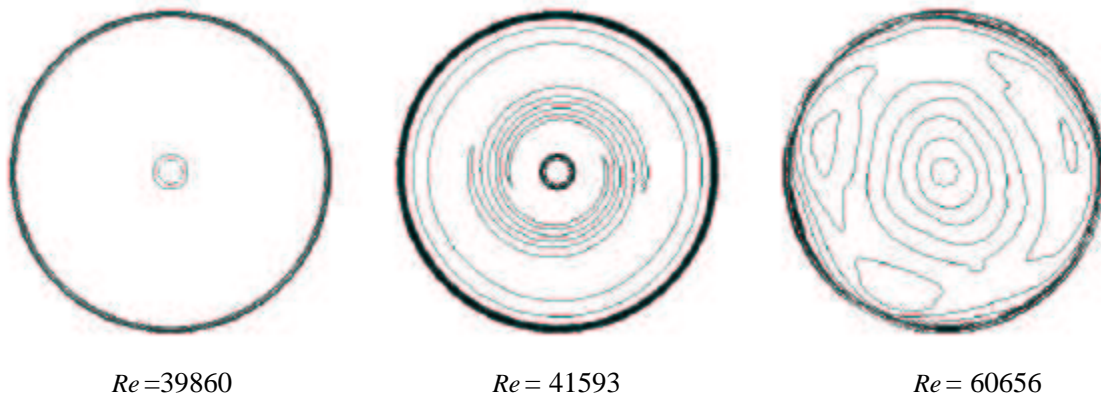


Fig.4 Iso-lines of the axial velocity component in the Ekman layer ($r_d = 0.954$, $z = 0.06$).



Fig.5 Variation of the enstrophy in the $r-z$ plane ($Re = 39860$).

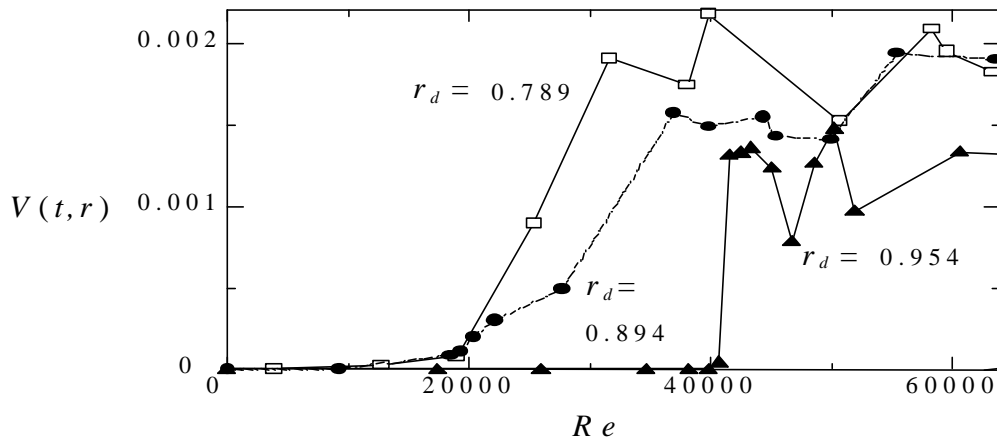


Fig.6 Variance $V(t, r)$ against the Reynolds number ($t=100.0$, $r =0.412$).

CONCLUSION

The flow around a rotating disc in a cylindrical enclosure is investigated numerically. The variance of the axial velocity component is a suitable measure of the axisymmetry and the transition to the asymmetric state of the rotating flow. Taylor-Couette flow develops in the clearance between the disc rim and the shroud of the enclosure, and it makes the flow on stationary lids unstable. The effect of the flow in the clearance becomes larger as the gap width increases. When the rim-shroud clearance is small, basic flow, spiral rolls and turbulent spiral rolls, which are observed in system with no clearance, are predicted, while spiral rolls tend to cease at larger clearance.

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