Very-Low-Frequency State in a Short Annulus Taylor-Couette Flow

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ABSTRACT

Taylor-Couette flow between two concentric rotating cylinders continues to provide a canonical physical system that has been instrumental in developments in nonlinear dynamics, routes to chaos, and equivariant dynamical systems (e.g. [1, 2, 3]). The study of the influence of endwalls and reflection symmetry at the annulus half-height [4, 5] opened up a new perspective into the importance of Z_2 symmetry (reflection) in Taylor-Couette flow, as well as in many other equivariant problems.

The impact of Z_2 symmetry is enhanced as the aspect ratio of the annulus is reduced [6]. Most theoretical and numerical studies of short annulus Taylor-Couette flow have been restricted to an axisymmetric subspace and have primarily considered steady Z_2-symmetry breaking. Experimentally however, very rich and complex spatiotemporal dynamics, including global bifurcations, have been observed that are intrinsically associated with the Z_2 equivariance of the system (e.g. [7, 8, 9, 10]). Equivariant bifurcation theory [11, 12, 13] provides a classification of possible bifurcation scenarios that may occur in the presence of symmetries. Two recent studies on short annulus Taylor-Couette flow with Z_2 reflection symmetry have identified two of the ways the basic state can become unstable, both breaking the Z_2 symmetry; in [14] the symmetry breaking is via steady-state pitchfork bifurcations and in [15] it is via a Hopf bifurcation. In both investigations, the analyses of the results were restricted to an axisymmetric $SO(2)$ subspace.

One of the main conclusions from [14] is the conjecture that at small aspect ratios (less than 0.5), the steady axisymmetric flow is unique, and yet they report observing time dependence and disorder at higher Reynolds number (Re) and claim that an understanding of the evolution remains an outstanding challenge. In this paper we investigate numerically the dynamics at aspect ratio $\Gamma = 0.5$ using a three-dimensional Navier-Stokes spectral solver. At small Re we found the steady-state pitchfork bifurcation that breaks the Z_2 symmetry, as well as the inverse pitchfork where the bifurcated states revert to a Z_2-symmetric state, as reported in [14]. At slightly higher Re, the flow becomes unstable via two Hopf bifurcations that break Z_2, one leading to an axisymmetric limit cycle, LC_0, and the other to a nonaxisymmetric rotating wave, RW_1.

The short aspect ratio of the annulus investigated here results in a basic state with a single outwards jet at the mid-height plane; much larger aspect ratios lead to states with Taylor vortex-like flows with multiple jets. From group theoretic considerations, we describe the four ways in which the basic state can become unstable via primary symmetry breaking bifurcations. Three of these are identified as solutions for this particular geometry. One is a steady axisymmetric pitchfork bifurcation, which has been previously investigated [14]. The other two are via Hopf bifurcations producing either an axisymmetric limit cycle or a rotating wave with azimuthal wave number $m = 1$, which are systematically investigated here for the first time. The two Hopf bifurcation curves intersect at a double Hopf bifurcation, at which point an unstable mixed mode also bifurcates. The role of this mixed mode is to either stabilize or destabilize the periodic solutions from the two Hopf bifurcations at Neimark-Sacker bifurcations. A second mixed mode, in the form of a modulated rotating wave (*MRW*), not directly associated with the double Hopf bifurcation, also bifurcates from the two periodic solutions at higher Reynolds numbers. It is stable (unstable) when it bifurcates from a stable (unstable) periodic state. These stable and unstable

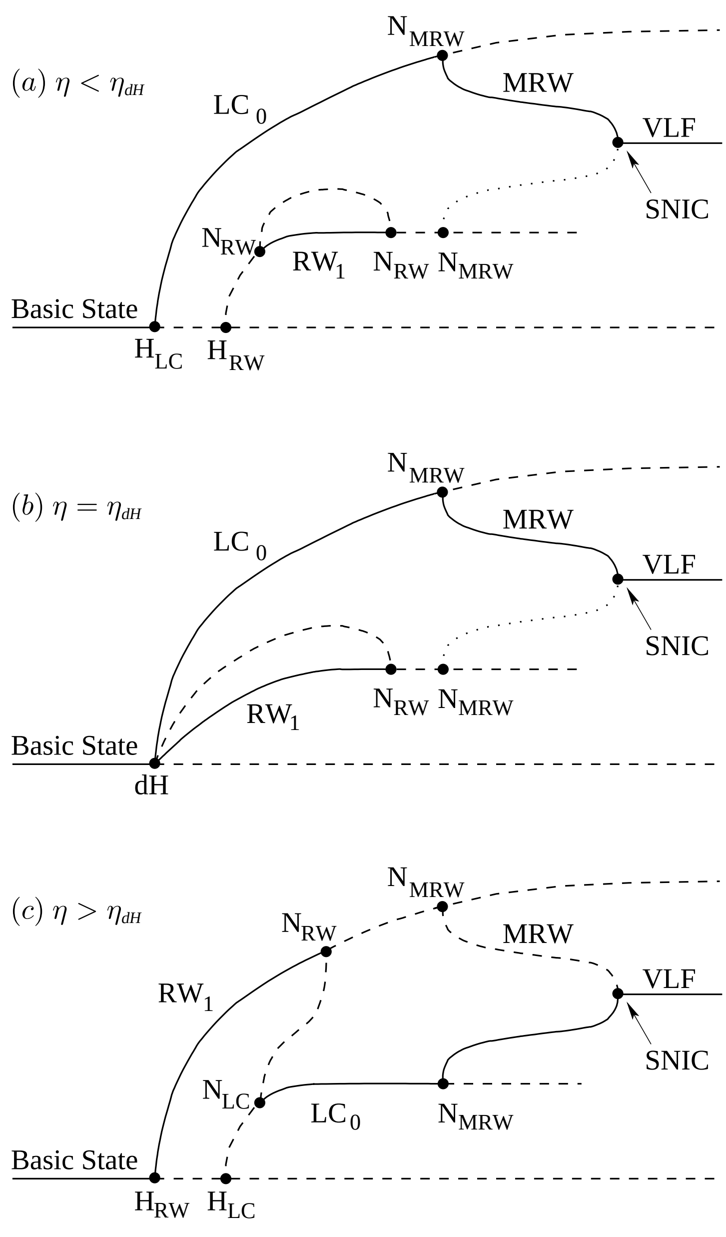

Figure 1: Schematic bifurcation diagrams, using Re as the bifurcations parameter, fixing $\Gamma = 0.5$ and for (a) $\eta < \eta_{dH}$, (b) $\eta = \eta_{dH}$ and (c) $\eta = \eta_d H$. H_{LC} and H_{RW} are Hopf bifurcations to LC_0 and RW_1, respectively, and dH is the codim-2 bifurcation where both H_{LC} and H_{RW} occur simultaneously. N_{LC}, N_{RW} and N_{MRW} are Neimark-Sacker bifurcations, and *SNIC* is the saddle-node-on-invariant-circle bifurcation leading to the *VLF* state.

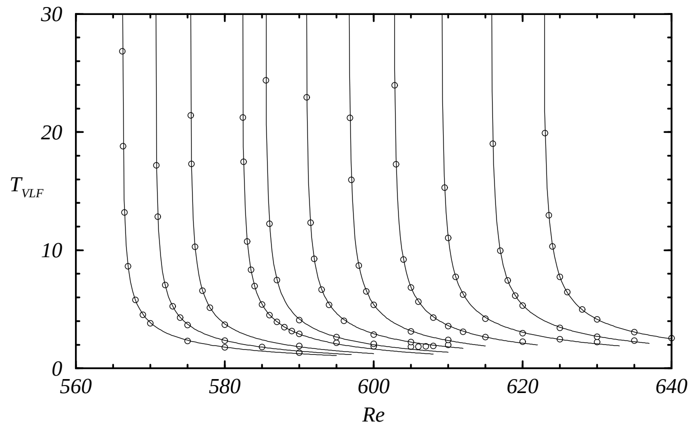

Figure 2: Variation of T_{VLF} with Re at $\Gamma = 0.5$ and $\eta = 0.650$, 0.655, 0.660, 0.667, 0.670, 0.675, 0.680, 0.685, 0.690, 0.695, 0.700; $T_{\mathit{VLF}} \to \infty$ at lower Re for lower η. The open circles are computed periods and the lines are fits of the form $T_{\mathit{VLF}} = a_0 + a_1/\sqrt{Re - Re_{\mathit{VLF}}}$.

modulated rotating waves are heteroclinically connected, and with increasing Reynolds number, collide and vanish in a saddle-node bifurcation. Following the collision, all that remains is an invariant manifold consisting of the previous heteroclinic connections. This invariant manifold is a stable three-torus, with two frequencies corresponding to those of the modulated rotating waves, and a third very low frequency that vanishes at the saddle-node-on-invariant-circle bifurcation (*SNIC*). This very-low-frequency state has been previously observed in experiments [6, 16], but here we present the first computed example of such a state, together with a comprehensive bifurcation sequence leading to its birth. The associated bifurcation diagrams are shown schematically in Fig. 1, and the third period, T_{VLF}, associated with *VLF*, is shown in Fig. 2 as a function of Re for $\Gamma = 0.5$ and various values of η, the radius ratio.

The very-low-frequency states have been observed experimentally in a number of Taylor-Couette flows where endwall effects are important, i.e. in short aspect ratio systems, although they have also been observed in experiments with aspect ratios of order 10. Also, the appearance of the *VLF* state has not only been associated with a *SNIC* bifurcation, but has also been associated with cycle-saddle homoclinic collisions in the same experiments but in different parameter regimes [16, 8]. The two global bifurcations are distinguished by the scaling law describing how the associated period, T_{VLF}, becomes unbounded. With the *SNIC* bifurcation, $T_{\mathit{VLF}} \sim 1/\sqrt{|Re - Re_{\mathit{VLF}}|}$, and for the homoclinic collision $T_{\mathit{VLF}} \sim 1/\log|Re - Re_{\mathit{VLF}}|$. Which is observed seems to depend on the path through parameter space taken. If the basic state first loses stability via a Hopf bifurcation where the resulting periodic state is set-wise Z_2-invariant, then it appears that the onset of a *VLF* state occurs via the *SNIC* bifurcation. If on the other hand, the basic state loses stability via a steady pitchfork bifurcation, producing two conjugate steady states that subsequently become unstable via Hopf bifurcations that result in a pair of limit cycles that are not Z_2-invariant, but are conjugate under a Z_2 reflection, then these may undergo a gluing bifurcation where the two cycles simultaneously collide homoclinically with the unstable (saddle) basic state. This gluing bifurcation produces a new Z_2-invariant cycle with an associated very-low-frequency that obeys a $\log|Re - Re_{\mathit{VLF}}|$ law. Such a gluing bifurcation in a short annulus Taylor-Couette experiment has recently been reported [10]. This all suggests that the two global bifurcations may be organized by a Takens-Bogdanov bifurcation at which a Hopf and a pitchfork bifurcation coincide. Recently, [17] has studied the dynamics associated with the normal form of a Takens-Bogdanov bifurcation with D_4 symmetry and has identified a scenario that spawns both types of global bifurcations. Although the symmetries of his problem differ from those of ours, there are sufficient features in common that suggest that the *VLF* states may well be organized by such a bifurcation. Investigations into this conjecture are currently underway.

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