Time-dependent solutions in rotating/non-rotating plane Couette flow

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ABSTRACT

We follow the time development of fluid motion resulting from oscillatory instabilities of the steady tertiary solution in the rotating plane Couette system. It is found that for a relatively small Reynolds number, Re, the time-dependent motion is characterised by a periodic motion with a single frequency. As the system rotation, Ω, varies the periodic motion undergoes a period doubling bifurcation.

Tracing the periodic solution in the $Re - \Omega$ space, we show that periodic motions exist even when Ω vanishes at a higher Re. They correspond to the periodic solutions for the non-rotating plane Couette system.

1 Background

Recently, Nagata(1998) investigated the stability of the tertiary solutions in rotating plane Couette flow with the result that the tertiary flows, which bifurcate from steady two-dimensional streamwise vortex flows, are stable within a certain interval of the system rotation, Ω, when the Reynolds number is relatively small, say $Re = 200$. The boundaries of the stability interval on the Ω axis are determined by perturbations which are subharmonic in the streamwise direction. As the Reynolds number is increased to $Re \simeq 250$, another type of instabilities begins to emerge in the middle of the stability interval at $\Omega \simeq 20$. These instabilities are characterised by an oscillatory nature. As Re is further increased, the oscillatory instabilities spread in the directions of both increasing and decreasing Ω, gradually contaminating the stability interval. It is expected that the tertiary flow is totally overtaken by time-dependent motions for large Reynolds numbers.

In this short note we examine the development of time-dependent motions in rotating and non-rotating plane Couette flow numerically.

2 Formulation

We consider a viscous incompressible fluid motion between two parallel plates separated by the distance L. The bottom plate moves along in its own plane with a constant speed $\frac{1}{2}U_0$ whereas the top plate moves in the opposite direction with the same speed. A constant spanwise rotation Ω_0 is imposed on the system. By using L as the length scale, L^2/ν as the time scale where ν is the kinematic viscocity, and ν/L as the velosity scale, we can express the nondimensional basic laminar flow as $U_B(z) = -Re\, z$ where z is the coordinate in the direction normal to the plates and Re is the Reynolds number defined by $Re = U_0 L/\nu$. For convenience we separate the velocity deviation $\boldsymbol{u}$ from the laminar state into the average part $\check{U}(z)$ and the residual

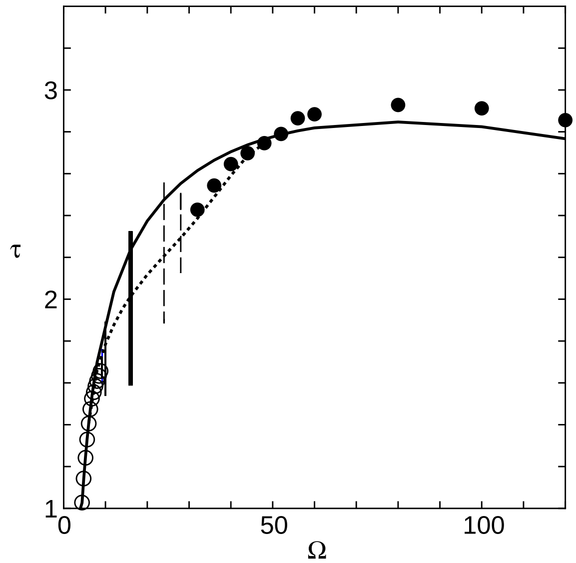

Figure 1: The momentum transport τ against the system rotation Ω for various types of motions at $Re = 400$.

$\check{\boldsymbol{u}}$. The nonlinear development of $\boldsymbol{u}$ is governed by two non-dimensional parameters: one is the Reynolds number and the other is the system rotation Ω defined by $\Omega = 2\Omega_0 L^2/\nu$ (see Nagata (1998)).

3 Method

In the present analysis we use three different numerical schemes as described below.

1. Newton-Raphson method for steady motions:

   The disturbance of a steady motion is expressed by the Fourier expansions in the streamwise and spanwise directions and the Chebyshev-polynomial expansions in the direction normal to the plates. The amplitudes of each disturbance component are computed by a Newton-Raphson iterative scheme. (The stability of steady motions is evaluated by applying Floquet theory.)

2. DNS:

   The time development of the disturbance is followed by a direct numerical simulation (DNS), where the time integration is performed on the full Navier-Stokes equation by using a pseudo-spectral method. The numerical scheme for the simulation is essentially the same as that used by Kim, Moin & Moser (1987).

3. Newton-Raphson method for periodic motions:

   We have enhanced Kawahara & Kida's (2001) iterative method for time-periodic solutions to implement Newton-Raphson computation of a fixed point in a Poincaré map. The Poincaré map, i.e. the one-period time integration of the Navier-Stokes equation, is computed by the DNS. The Jacobian matrix is evaluated by a finite-difference approximation. We first employ a stable periodic state, which is accesible to the DNS, as an initial guess

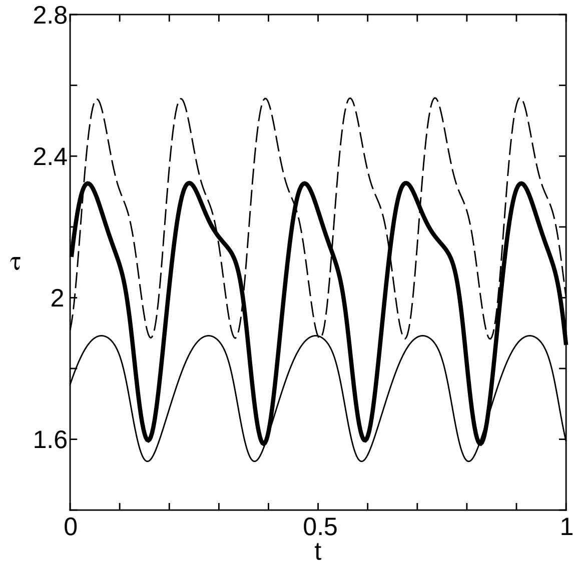

Figure 2: The time development of the momentum transport τ for three periodic motions at $Re = 400$.

for the Newton–Raphson iteration to extend a periodic solution to an unstable region in parameter space.

4 Results

The figure 1 shows the variation of the momentum transport, $\tau \equiv [U_0(\pm\frac{1}{2})+\check{U}(\pm\frac{1}{2})]'/[U_0(\pm\frac{1}{2})]'$, against Ω for various types of motion when $Re = 400$. The solid curve indicates the steady three-dimensional solution with the streamwise wavenumber $\alpha = 1.0$ and the spanwise wavenumber $\beta = 3.117$, whereas the dotted curve indicates the streamwise vortex flow with $\alpha = 0$ and $\beta = 3.117$. They are obtained by the Newton-Raphson method. The vertical lines indicate a single periodic motion (the thin solid lines for the smaller Ω side and the thin dashed lines for the larger Ω side) and a doubly periodic motion (the thick solid line) obtained by the DNS. Also obtained by the DNS are steady flows indicated by open circles for the larger Ω and closed circles for smaller Ω.

We see that the streamwise vortex flow which bifurcates from the basic flow at $\Omega_{\rm C} = 4.3$, becomes unstable to three-dimensional perturbations as Ω is increased and the stability is taken over by the steady three-dimensional flow at $\Omega_1 = 6.5$. The stable steady three-dimensional flow lasts over a small range of Ω and a periodic motion due to a Hopf bifurcation sets in at $\Omega_{\rm H1} = 8.4$. The time development of the momentum transport for the periodic motion at $\Omega = 10$ is shown by the thin curve in Fugire 2, confirming that the periodic motion has a single period $T = 0.227$. As the system rotation is increased to $\Omega = 16$ the period of the motion becomes doubled as indicated by the thick curve in Figure 2. When the system rotation is increased to and over $\Omega = 24$ the motion becomes single periodic again as indicated by the dashed curve in Figure 2, until the periodic motion shrinks to a steady three-dimensional flow at $\Omega_{\rm H2} = 32$. The three-dimensionality vanishes at $\Omega_2 = 47$ and the streamwise vortex flow is obtained for larger Ω values and finally the basic state is recovered although it is not shown in Figures 1 and 2.

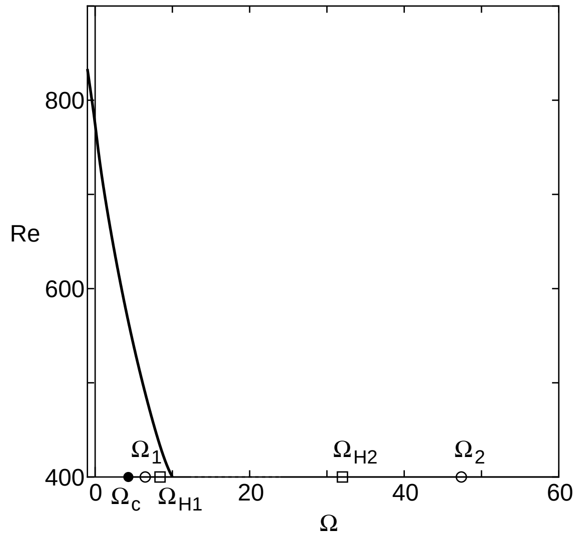

Figure 3: The trace of the periodic solution with the period $T = 0.227$ in the $\Omega - Re$ space.

Starting from the periodic solution detected at $(\Omega, Re) = (10, 400)$ we trace the periodic motion in the $\Omega - Re$ space by keeping the period T constant (i.e. $T = 0.227$) as indicated by the curve in Figure 3. The periodic motions on the curve are obtained by applying the Nweton-Raphson iterative scheme to find a fixed point in the Poincaré map. As we can see the curve extends to a smaller Ω, crossing the line of $\Omega = 0$ at $Re = 774$. The solution at the intersection of the trace and the line of $\Omega = 0$ corresponds to the periodic motion for non-rotating plane Couette system.

5 Summary

We have shown that the periodic motions, which bifurcate from the tertiary flow in the rotating plane Couette system, undergo a period doubling bifurcation. We have traced the periodic motions in the parameter space and found that they exist even when the system rotation is absent.

Preliminary investigation has indicated that the periodic motions obtained in the present paper for non-rotating plane Couette system do not have connection with those periodic motions found by Clever & Busse (1997) or Kawahara & Kida (2001).

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