Anomalous dispersion of Couette-Taylor spirals

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ABSTRACT

Competition of spirals in Couette – Taylor system between counter rotating cylinders leads, for small supercriticality to the formation of localized source. Measuring the group velocity as function of the amplitude, we have determined that waves forming such source have anomalous dispersion, in the sense that phase and linear group velocity of each have opposite signs.

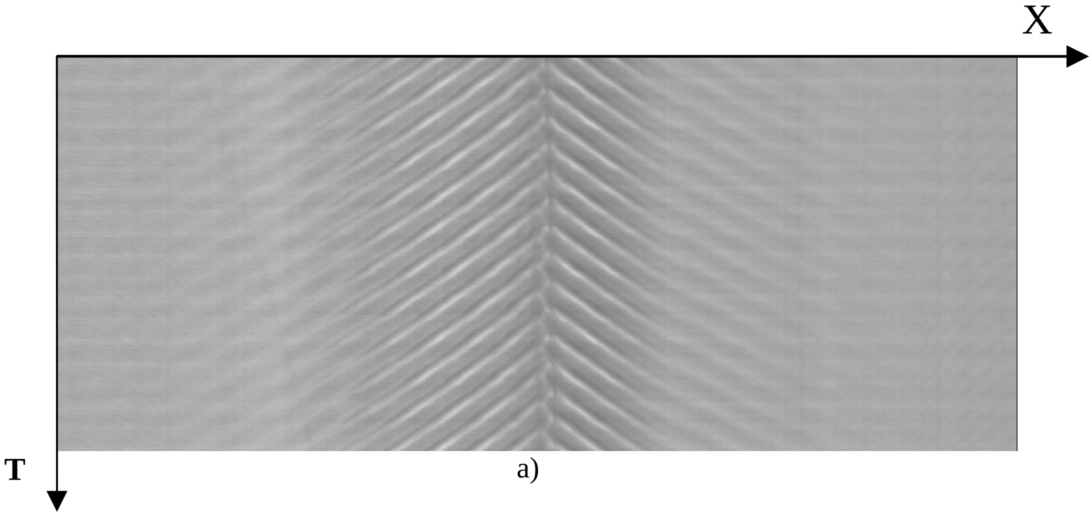
Our Couette - Taylor system has following characteristics: the inner cylinder with the radius $a = 4.459$ cm was made of black Delrin, the outer cylinder was made of transparent Plexiglas and its internal radius was $b = 5.050$ so the gap between cylinders is $d = b - a = 0.591$ cm and the working length of the system is $L = 27.5$ cm. So, the experiment was performed in a spatially extended system with an aspect ratio $\Gamma = L/d = 46.5$, and radius ratio $\eta = a/b = 0.883$. Cylinders were rotated independently by two DC motors. Thus in this Couette - Taylor system the flow control parameters are Reynolds numbers defined for the inner and outer cylinder respectively : $R_o = \Omega_o bd/\nu$ and $R_i = \Omega_i ad/\nu$ where Ω_o, Ω_i are the angular frequencies of outer and inner cylinders respectively, ν is for kinematic viscosity of fluid. We have used distilled water ($\nu = 10^{-2}\, cm^2/s$ at $T = 21^0 C$) with 2% volume of flakes of Kalliroscope AQ1000 for the visualization of flow structures.

To obtain information on space – time behavior of flow, we have used a linear 1024–pixel charge coupled device (CCD) array oriented along the cylinders axis to record instantaneous intensity distribution $I(X)$ of light reflected by Kalliroscope.

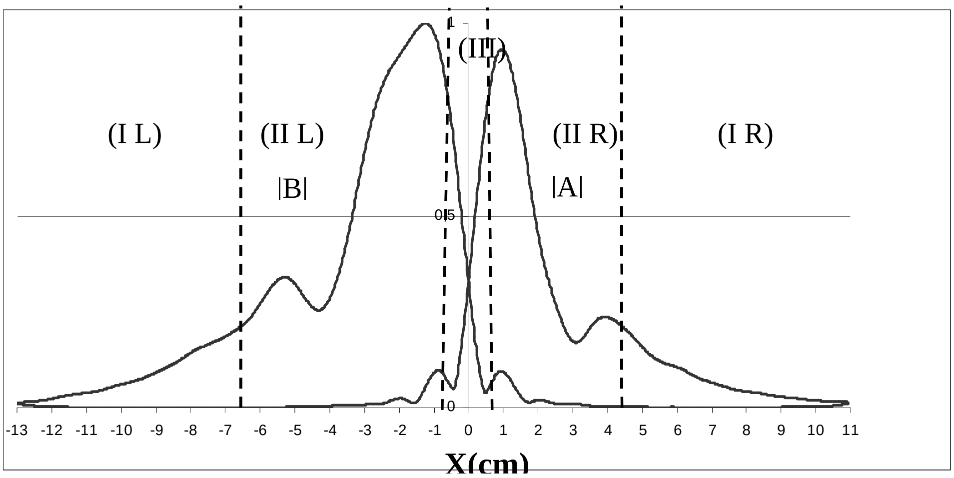
The recorded length was from 20 to 25 cm in the central part of the system, corresponding to the a spatial resolution from 50 to 40 pixel/cm. The intensity is sampled in 256 values, displayed in gray level time interval along time axis to produce space – time diagrams $I(X,T)$ of the pattern. To extract the spatial and temporal properties we have performed the complex demodulation technique of the signal [1] which allows in particulary, to separate the right traveling and left traveling waves and to represent them as $|A(X,T)|\exp[i(k_A x - \omega_A t)]$ and $|B(X,T)|\exp[i(k_B x - \omega_B t)]$.

The phase and group velocity of waves are calculated from the relation $v_{ph} = \omega/k$ and $v_g = d\omega/d\kappa$. We may choose the length scale as d, the time scale as $\tau = d^2/\nu$, leading to dimensionless wavenumber and frequency $q = kd$ and $\Omega = \omega d^2/\nu$. The velocities are scaled by the characteristic diffusion velocity ν/d.

We investigated regimes of spirals which we have observed as a first supercritical instability mode from the base flow in a case of counter-rotating cylinders [2] for $180 < R_i < 500$ and $-1155 < R_o < -155$. A typical space-time diagram shown in the Figure 1 exhibits very clearly the position of phase fronts of wave disturbances corresponding to right and left traveling spirals.



a)



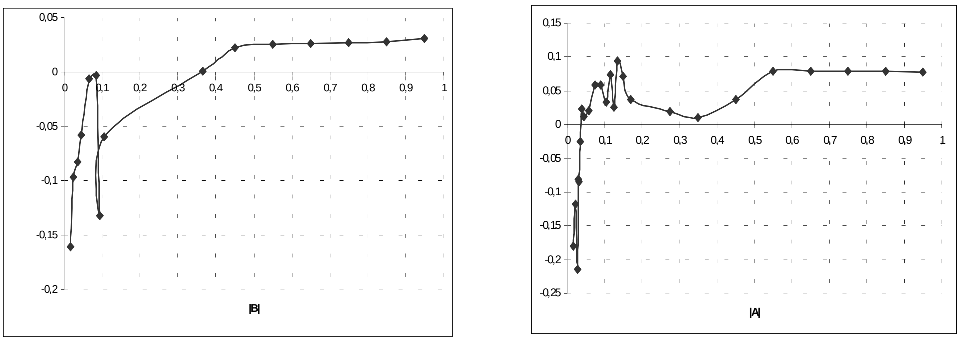
b)

Figure 1 : a) Space – time diagram and b) spatial profile of the averaged amplitude for counter-propagating spirals propagating for $R_o = 622$, $R_i = -341$($\varepsilon = 0.015$).

The curves of axial variation of time averaged amplitude of left and right traveling waves (Fig.1-b) exhibit three distinct zones : two zones where there is only one wave (I L,R) and a zone of interaction (II L, R). The zones (I L,R) are characterized by small amplitudes (0 < A < 0.1 A_{max}), and the zones (II L,R) are characterized by large amplitudes (0.1A_{max}<A <A_{max}). In the strongly interacting region of left and right wave (III), the amplitude decreases rapidly. The localized distribution of amplitudes of counter propagating waves was found for small supercriticalities (ε < 0.03). The local depressions in the averaged amplitude profiles correspond to the presence of defects in the pattern.

The counterpropagating waves have the same wavenumber but slightly different frequencies which are dependent on spatial coordinate. Near the source, the wavenumber and frequency pertain a strong change due to phase jump. The right traveling wave (for x > 0) has an average frequency $\Omega_R = 62.31$ (equivalent $\omega_R = 1.64\,Hz$) different from that of left traveling wave (for x < 0 : $\Omega_L = 59.21$ equivalent $\omega_L = 1.56\,Hz$). The wanumber is almost constant in the zone of interaction (III), it decreases linearly in the zones (II L,R) where there is only one wave, and in the zones (I L, R) it is constant within spatial experimental resolution. The local depressions in the wavenumber profiles correspond to the presence of defects in the pattern.

Using the spatial variation of wave parameters (amplitude, wave frequencies, wave numbers), it is possible to plot the curve of dispersion in different region for different amplitude interval separately for left and right wave (Fig.2).The dispersion curve $\Omega = f(q)$ in different zones for amplitude allows for computation of the group velocity $V_g = d\Omega/dq$ as a function of the amplitude for left and right wave separately. We have found that the group velocity of the right traveling spiral is positive for large amplitude while it is negative for small amplitude $0 < A < 0.05A_{max}$; the group velocity of left travelling spiral is negative for large amplitude and positive for small amplitude $0 < B < 0.35B_{max}$. We therefore have anomalous dispersion for small amplitudes and normal dispersion for large amplitudes.



a) b)

Figure 2 : Group velocity dependence on amplitude : a) for left and b) right spiral

The field of spiral pattern can be described by [3-6]

$$U = A(T,X)e^{i(-qX+ m\theta + \Omega T)} + B(T,X)e^{i(qX+m\theta+ \Omega T)} + c.c. \qquad (1)$$

where Ω is the bifurcation Hopf frequency, m is the azimuthal integer wavenumber, c.c. stands for complex conjugate. The amplitudes of right and left traveling spirals $A(T,X)$ and $B(T,X)$ satisfy the coupled Ginzburg-Landau equations [7-9]:

$$\tau_0\left(\frac{\partial A}{\partial T}+V_{g,R}\frac{\partial A}{\partial X}\right)=\varepsilon(1+ic_0)A+\xi_0^2(1+ic_1)\frac{\partial^2 A}{\partial X^2}-g(1+ic_2)|A|^2A-d(1+ic_3)|B|^2A \qquad \text{(2-a)}$$

$$\tau_0\left(\frac{\partial B}{\partial T}-V_{g,L}\frac{\partial B}{\partial X}\right)=\varepsilon(1+ic_0)B+\xi_0^2(1+ic_1)\frac{\partial^2 B}{\partial X^2}-g(1+ic_2)|B|^2B-d(1+ic_3)|A|^2B \qquad \text{(2-b)}$$

Here τ_0 is the characteristic time, ξ_0 is the characteristic length, $V_{gR,L}$ represent linear group velocity of right and left spirals, $c_{0,1}$ are linear dispersion coefficients $c_{2,3}$ are coefficients of nonlinear dispersion coefficients which leads to a shift of frequency, d expresses the mutual suppression of the modes.

Different theoretical analysis of equations (2) have shown that sources and sinks occur when the coupling ceofficient d >g, in this case, each mode suppresses the other [7-9]. The flow pattern tends to form domains of either left or right traveling waves separated by domains walls. If one assumes that the linear group velocity entering the system of equations (2) V_g is negative, then one finds that the front between counterpropagating spirals observed in our experiment is a source. Moreover, some of the constants of the pattern (linear group velocity, characteristic time τ_0, the ratio d/g) can be determined from comparison of approximate theoretical solutions and experimental profiles. In particular, it is found that in the linear growth regime, the group velocity has a sign opposite to that of the phase velocity for each left and right wave respectively.

We have shown that the supercritical spiral pattern in counter-rotating Couette-Taylor system exhibits a spatial variation with anomalous dispersion for small amplitude regions and normal dispersion for large amplitude and that left and right traveling spirals are separated by a stable source. The anomalous dispersion can be explained within the framework of the coupled Ginzburg-Landau equations for counterpropagating waves.

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