Reversing and non-reversing modulated Taylor-Couette flow

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ABSTRACT

The classical formulation of the Taylor-Couette flow problem consists of an incompressible fluid of constant kinematic viscosity ν which is contained between two coaxial cylinders; the outer cylinder is held fixed and the inner cylinder rotates at constant angular velocity Ω_1. Our concern is the case in which Ω_1 is not constant, but oscillates harmonically in time. The oscillating boundary induces a damped viscous wave which penetrates into the fluid a distance of the order of the thickness of the Stokes layer, $\delta_s = (2\nu/\omega)^{1/2}$, where ω is the frequency of modulation. We are concerned with the case in which ω is low enough that δ_s is comparable to δ, where δ is the gap width. Unlike previous investigations, our modulation has zero mean, $\Omega_1(t) = \Omega_{1a}\cos(\omega t)$, as in the motion of a washing machine.

Formulation of the problem

We use cylindrical coordinates (r, θ, z), call R_1 and R_2 the inner and outer radius respectively and make the usual assumption that the cylinders have infinite height. The governing equation is

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = -\frac{1}{\rho}\boldsymbol{\nabla} p + \nu\nabla^2\boldsymbol{v}, \tag{1}$$

with $\boldsymbol{\nabla}\cdot\boldsymbol{v} = 0$ and boundary conditions $v_r = v_\theta = v_z = 0$ at $r = R_2$, and $v_r = v_z = 0, v_\theta = R_1\Omega_1(t)$ at $r = R_1$. We make the equation dimensionless using the length scale $\delta = R_2 - R_1$ and the viscous time scale δ^2/ν and introduce the Reynolds number

$$Re_1(t) = Re_{mod}\cos(\omega t), \tag{2}$$

where $Re_{mod} = \Omega_{1a}R_1\delta/\nu$ and now t and ω are dimensionless. The other parameter of the problem is the radius ratio $\eta = R_1/R_2$. We call $Re_{10} = \Omega_{10}R_1\delta/\nu$ the Reynolds number which corresponds to the onset of Taylor vortex flow in the steady case.

Equation (1) is time-stepped using a combination of second order accurate Crank-Nicolson and Adams-Bashforth methods. The velocity components are represented by potentials which are expanded spectrally over Fourier modes in the azimuthal and axial directions and over Chebyshev polynomials in the radial direction. The code was tested in the axisymmetric and wavy regimes against published results.

Results

We assume $\eta = 0.75$ throughout, for which, in the steady case, $Re_{10} = 85.78$ and the (dimensionless) critical axial wavenumber is $\alpha_c = 3.13$. All our calculations were performed with variable α to determine the critical axial wavenumber at each frequency. We have also performed fully three-dimensional calculations that allow the existence of non-axisymmetric wavy modes, but it was found that, within the parameter range explored, the wavy modes always decay, and the resulting solution is axisymmetric. Initial conditions consist of seeding all spectral modes with random numbers of order of magnitude 10^{-3}; we then integrate the equations in time, until, after an initial transient, a settled oscillation is achieved.

Typical results at lower frequency of modulation are shown in figure 1.

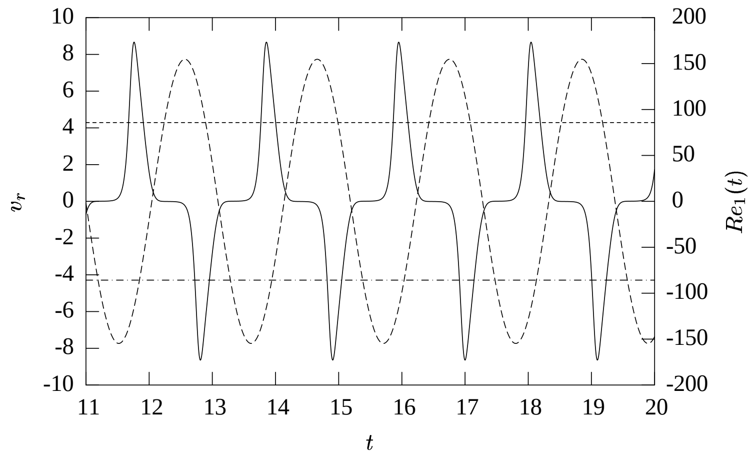

Figure 1: Radial velocity v_r versus time in the middle of the gap at the outflow computed at the (dimensionless) position $z = \pi/\alpha$, $r = (1+\eta)/2(1-\eta)$ for RTVF. 12, 8, 4 radial, axial, azimuthal modes respectively, $\eta = 0.75$, $Re_{mod} = 154.71$ (which is $Re_{mod} = 1.1 Re_{mod,c}$ with $Re_{mod,c} = 140.65$), $\omega = 3$, $\alpha_c = 2.86$. Horizontal lines show $\pm Re_{10} = 85.78$ and the dashed curve is $Re_1(t)$.

The solid curve represents the radial velocity component $v_r(t)$ computed at the outflow jet ($z = \pi/\alpha$) in the middle of the gap, which is zero when the flow is purely azimuthal (circular Couette flow). The dashed curve represents the driving Reynolds number $Re_1(t)$. The horizontal line at $Re_1 = Re_{10}$ in the steady case denotes the onset of Taylor vortex flow. The second horizontal line at $Re_1 = -Re_{10}$ corresponds to the onset of Taylor vortex flow of opposite polarity (created when the cylinder rotates in the opposite direction). Initially, the Reynolds number $Re_1(t)$ increases starting from the left of figure 1. Quasi-statically, we expect that, when $Re_1(t)$ reaches a value of the order of Re_{10}, azimuthal flow becomes unstable and v_r grows exponentially; then, as $Re_1(t)$ becomes smaller than Re_{10}, v_r peaks and drops quickly toward zero. The phase lag between the maxima values of $Re_1(t)$ and v_r is expected, as it takes a certain time for the fluid in the middle of the gap to respond to the drive. Soon afterward the motion of the inner cylinder becomes supercritical again but in the opposite direction, and a new Taylor vortex pair is formed starting from the vanishingly small remains of the previous cycle. Note that this time the flow has opposite polarity, so v_r is negative (inflow jet). We call this flow reversing Taylor vortex flow (RTVF).

Figure 2 shows typical results at higher frequency of modulation.

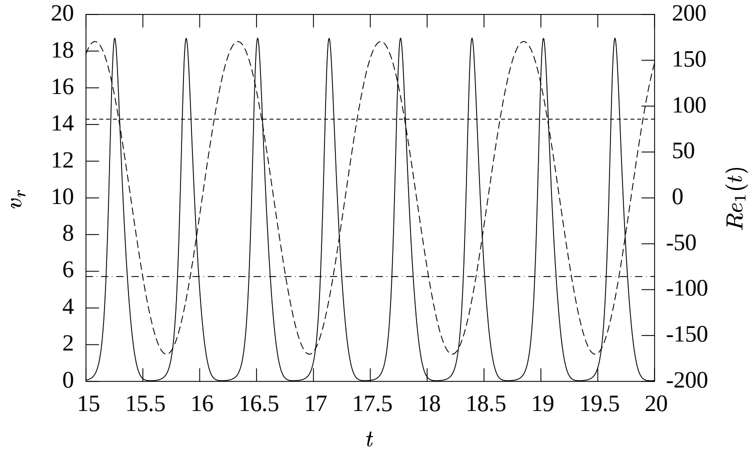

Figure 2: Radial velocity v_r versus time in the middle of the gap at the outflow computed again at the (dimensionless) position $z = \pi/\alpha$, $r = (1+\eta)/2(1-\eta)$ for NRTVF. Parameters as in figure 1 except $Re_{mod} = 170.41$ (which is $Re_{mod} = 1.1 Re_{mod,c}$ with $Re_{mod,c} = 154.91$), $\omega = 5$, $\alpha = 3.73$.

It is apparent that the direction of the radial velocity v_r remains the same (the peaks are always positive), despite the change of direction of the driving inner cylinder. We call this flow non-reversing Taylor vortex flow (NRTVF).

By examining v_r at the symmetric point plotted and also at other non-symmetric points we see that both RTVF and NRTVF are synchronous with NRTVF being a harmonic of the imposed driving Reynolds number with frequency twice the driving frequency.

Figure 3 shows how the Reynolds number Re_{mod} depends on the frequency ω. Each point on the figure represents the result of a separate run of the code starting from seed with the axial expansions containing multiples of the critical wavenumber. For lower frequencies RTVF is the first to onset, whereas, for higher frequencies NRTVF is the first.

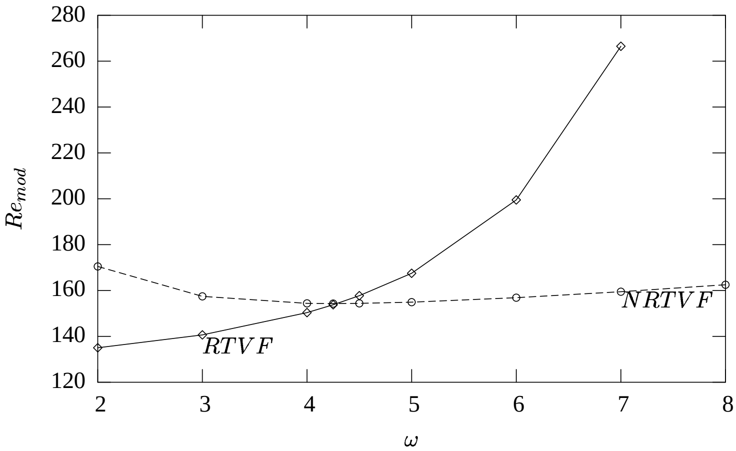

Figure 3: Critical Re_{mod} of the inner cylinder versus frequency ω for RTVF and NRTVF.

Previous work on modulated Couette flow was done by Carmi & Tustaniwskyj (1981) who did not detect the existence of NRTVF – their approach was via Floquet theory. The fact that v_r in RTVF becomes negative, but only decays to a certain order of magnitude level in NRTVF suggests that NRTVF maybe due to finite-amplitude effects. However, Lopez & Marques (2002) have found reversing and non-reversing solutions for modulation of the outer cylinder, using Floquet analysis. This suggests that NRTVF may be due to a linear instability too.

Conclusion

We have found that, if the amplitude of the modulation is large enough to destabilize circular Couette flow, two classes of Taylor vortex flow are possible: reversing and non-reversing, and both solutions are axisymmetric. In the latter the Taylor vortex pairs always rotate in the same direction, despite the inner cylinder driving the flow in the opposite direction. Vortices whose meridional circulation is not affected by the direction of the basic azimuthal flow have also been noticed in recent numerical simulations of time-modulated spherical Couette flow by Zhang (2002) and Zhang & Zhang (2002).

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