Thermocapillary-buoyant convection in laterally heated liquid layers

Santiago Madruga ([1]) ([2]), Carlos Pérez-García ([1])([3]) *, Georgy Lebon([2])

([1]) Instituto de Física, Universidad de Navarra, E-31080 Pamplona, Navarra, Spain.
e-mail: carlos@fisica.unav.es

([2]) Departement AGO, Liège University, B5, Sart-Tilman, B-4000 Liège, Belgium.

([3]) Depto. Ingen. Mecánica, E. T. S. Ingen. (TECNUN), Univ. Navarra, E-20018 San Sebastián, Spain.

ABSTRACT

Buoyant-thermocapillary convection is a fluid motion driven by buoyancy effect and temperature-induced interfacial-tension variation. Such kind of flow can arise in a liquid-gas or a liquid-liquid interface subjected to a temperature gradient. For laterally-heated systems the instability mechanism is quite complex. A *basic flow* settles down in the fluid as soon as a lateral heating is applied. Smith and Davis [1] discussed this problem by considering only thermocapillary flows. They showed that for intermediate and low Prandtl numbers traveling oblique waves, known as *hydrothermal waves* superpose to the basic flow beyond a given temperature gradient threshold [2]. Later on Mercier and Normand [3] extended these calculations by taking into account buoyancy effects and thermal transfer properties at the interface [3]. Daviaud and Vince [4] reported the first observation of hydrothermal waves in a shallow layer of silicon oil of $0,65cSt$ (Pr = 10). Theoretical calculations by Priede and Gerbeth [5] showed that, in order to understand the distinct regimes encountered in experiments by Riley and Neitzel [6] it is important to distinguish between *absolute and convective* instabilities. Recently Pelacho *et al.* [7] and Burguete *et al* [8] reported new results on hydrothermal waves in different geometries. The first aim of this communication is to complete the linear stability analysis for a liquid with $P_r = 10$ using the concept of convective/absolute instabilities and to compare these results with the experimental findings in [7],[8]. We find a better agreement between the theoretical predictions for the hydrothermal waves and the experiments than found until now using the more conventional convective theory.

A common simplification in dealing with thermocapillary flows consists in considering convection solely in the liquid layer, while the effects of the upper gas are accounted for through the Biot number Bi, an empirical parameter characterizing the conductive heat transfer. To justify this approximation a full hydrodynamics analysis of the two–layer system would be required. As showed by Engel and Swift [9] two liquid layers under vertical thermal gradients display interesting oscillatory phenomena as showed by Engel and Swift [9]. The second aim of this communication is to analyze the stability of a two-layer system with a horizontal temperature gradient. We consider the three pair of liquids used in convective experiments by Molteno *et al.* [10] and Juel *et al.* [11]. Four different velocity flows and temperature profiles are found for the basic state. A linear analysis with respect to two and three dimensional perturbations reveals the existence of three kind of patterns. Depending on the height of both liquids several situations may occur as 1) *waves propagating from cold to hot*, 2) *waves propagating from hot to cold* and 3) *stationary longitudinal rolls*. The instability mechanisms are discussed and a qualitative interpretation of the different behaviors exhibited by the system is provided. In some configurations it is possible to find a codimension-two ($cod - 2$) point created by the interaction of two Hopf modes with different frequency and wavenumber. These features suggest that the two-liquid layers system may be an interesting arena to study the propagation and interaction of hydrothermal waves.

REFERENCES

[1] M.K. Smith and S.H. Davis, *Instabilities of dynamic thermocapillary liquid layers. Part 1. Convective instabilities.* J. Fluid Mech., **132**, 119 (1983).

[2] M.K. Smith, *Instability mechanisms in dynamic thermocapillary liquid layers.* Phys. Fluids, **29**, 10 (1986).

[3] J.F. Mercier and C. Normand, *Buoyant-thermocapillary instabilities of differentially heated liquid layers.* Phys. Fluids, **8**, 1433 (1996).

[4] F. Daviaud and J.M. Vince *Traveling waves in a fluid layer subjected to a horizontal temperature gradient*, Phys. Rev. E, **48**, 4432 (1993).

[5] J. Priede and G. Gerbeth, *Convective, absolute, and global instabilities of thermocapillary-buoyancy convection in extended layers* Phys. Rev. E, **56**, 4187 (1997).

[6] R. J. Riley and G.P. Neitzel, *Instability of Thermocapillary-Buoyancy Convection in Shallow Layers. Part 1. Characterization of Steady and Oscillatory Instabilities.* J. Fluid Mech., **359**, 143 (1998).

[7] M.A. Pelacho, A. Garcimartn, and J. Burguete *Local Marangoni number at the onset of hydrothermal waves*, Phys. Rev. E, **62**, 477 (2000).

[8] J. Burguete, N. Mukolobwiez, F. Daviaud, N. Garnier, and A. Chiffaudel *Buoyant-thermocapillary instabilities in an extended liquid layer subjected to a horizonal temperature gradient*, Phys. Fluids **13**, 2773 (2001).

[9] A. Engel and J.B. Swift, *Planform selection in two-layer Bénard-Marangoni convection.* Phys. Rev. E, **62**, 6540 (2000).

[10] W.A. Tokaruk, T.C.A. Molteno, and S.W. Morris, *Bénard-Marangoni convection in two layered liquids.* Phys. Rev. Lett., **84**, 3590 (2000).

[11] A. Juel, J.M. Burgess, W.D. McCormick, J.B. Swift, and H.L. Swinney, *Surface tension-driven convection patterns in two liquid layers.* Physica D, **143**, 169-186 (2000).