

Transition to chaotic patterns in Rayleigh-Bénard convection in rotating cylinders

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ABSTRACT

In rotating Rayleigh Bénard convection, Coriolis force stabilizes the conductive state, and the convective onset increases as rotation increases. The strength of the Coriolis force is represented by the dimensionless rotation rate $\Omega = 2\pi f d^2/\nu$ where f is the rotation frequency. The conductive state becomes unstable to stationary convective parallel rolls as in Rayleigh Bénard convection, but overstability is also possible for small Prandtl numbers. In the nonlinear regime, one of the most interesting phenomena is the Küppers-Lortz instability. This instability has been the subject of many theoretical and experimental works mainly because the primary bifurcation is a direct transition from the conductive regime to a chaotic state (spatio-temporal chaos). Rigid boundary conditions have a strong effect on patterns, both in perturbing solutions of the infinite system and in selecting particular solutions. Besides the stationary and oscillatory rolls expected in infinite layers, experiments and linear stability analyses have shown that in a rotating cylindrical (finite) layer of fluid the conductive state may be also unstable to rotating travelling waves, side wall attached or spiral body modes. This side wall mode instability may set in at lower values of the Rayleigh number than the critical value corresponding to the parallel rolls expected for infinite rotating layers of fluids. The stability boundaries of travelling waves and bulk waves depend on the aspect ratio ($\Gamma =$ radius-to-height ratio) and on the rotation rate of the cylinder (Goldstein et al. 1993). An asymptotic analysis in the limit of high rotation rate by Herrmann & Busse (1993) and by Kuo and Cross (1993) predicts that the onset of convection in the form of sidewall travelling walls grows as Ω and also that the wave propagates against the sense of rotation inside the cylindrical wall and with the sense of rotation if the fluid is outside the wall.

We present here the numerical solutions of the equations based on a pseudo-spectral collocation-Chebyshev expansion in both non-homogeneous radial and axial directions (r, z) and based on the 2π - periodicity of the solution in this configuration, a Fourier-Galerkin method is used in the azimuthal direction. The difficulty at the axis ($r=0$) has been avoided with a variable transformation by multiplying all the dependent variables by r (Serre & Pulicani 2001). The velocity-pressure coupling, has been overcome by the use of an improved projection scheme for time discretization (Serre & Pulicani 2001) using a pressure predictor computed at each time step. The time integration scheme is semi-implicit second-order accurate. It corresponds to a combination of the second-order Euler backward differentiation formula and the Adams-Bashforth scheme for the non-linear terms. The computations presented here were performed for confined layers of fluid with Prandtl number $\sigma = 5.3$. The spatio-temporal behaviour of the convective flow has been studied in two geometrical configurations, rotating cylinders of circular cross section and of annular cross section. The values of the rotation rates are intermediate, $\Omega = 60$ and $\Omega = 180$.

In the numerical results the first unstable mode observed in the circular cross section cavity is the slow body travelling wave, for both insulating and conducting sidewall thermal boundary conditions. The bulk travelling waves have not been obtained in the annular cross section cavities, since the bulk spiral patterns do not fit the annular cavity. In the transition to chaos we have found several nonlinear regimes depending on the geometry and on the thermal boundary conditions.

Bulk and sidewall travelling waves - The convective flow obtained in a circular cell of $\Gamma = 5$, rotating at $\Omega = 60$ with insulating thermal boundary conditions at the sidewall is presented in Figure 1. The

critical Rayleigh number Ra_c is in the interval $5300 < Ra_c < 5400$.

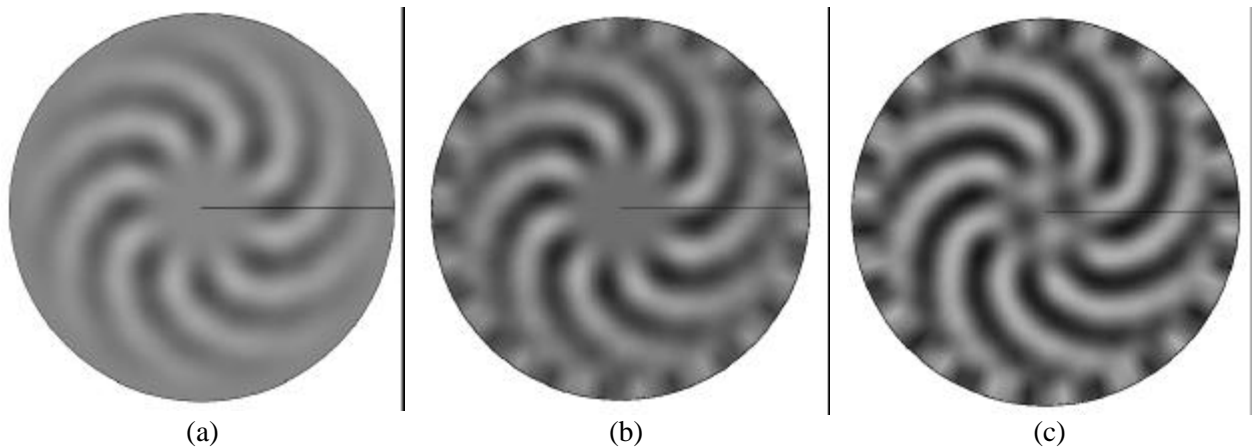
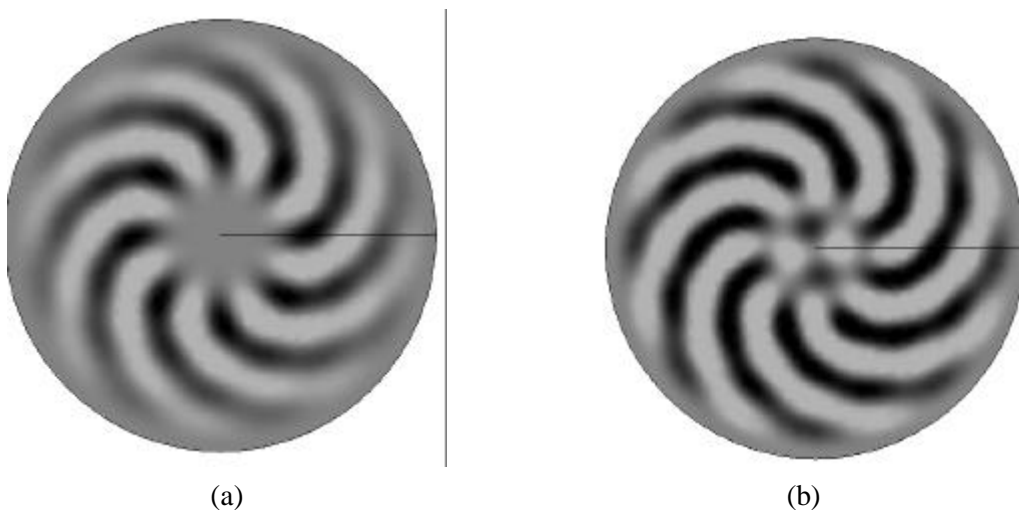


Figure 1: Iso-lines of temperature at mid-height in a cylinder $\Gamma=5$, $W=60$ (insulating thermal boundary condition). (a) $Ra=5400$, (b) $Ra=5600$, (c) $Ra=6000$

At $Ra=5400$, the flow is oscillatory. A rotating wave is observed in the inner part of the cylinder. The flow pattern is a spiral convection structure with 8 arms which, under the effect of the external rotation, precesses in the prograde direction with respect to the rotating frame of reference (Figure 1a) (dark regions are warm upflows and bright regions are cold downflows). The low value of the angular precession frequency indicates that this spiral pattern is related to the slow rotating wave predicted by linear stability (Goldstein et al. 1993). Increasing Rayleigh number to $Ra=5600$, a new set of azimuthally periodic travelling rolls is observed near the sidewall (Figures 1b and c). The frequency is about 8 times larger than the frequency associated with the spiral structure in the bulk. The slow rotating wave in the bulk and the fast wall mode rotate in opposite direction. The time behaviour depends on the location: near the wall the time signal is quasiperiodic with two frequencies (those of the fast wall mode and the slow bulk mode; in the bulk the time signal is oscillatory, but near the axis the time signal is steady. These modes are then spatially separated and the wall rotating wave has no effect on bulk convection at the middle of the cylinder. At $Ra=6000$, two protuberances occur at the centre of the cavity. The time behaviour is different in the three flow regions: oscillatory within the bulk, as in the previous Ra , but in the near axis region the time behaviour is now chaotic.

Bulk travelling wave and chaotic roll breaking - The flow was investigated in a circular cell with conducting thermal boundary conditions at the sidewall. The transition from an initially stationary conductive state to an oscillatory motion was observed in a range of Ra $5300 < Ra_c < 5400$, similarly to the preceding case. The evolution of the flow pattern with increasing Rayleigh number is shown in Figure 2.



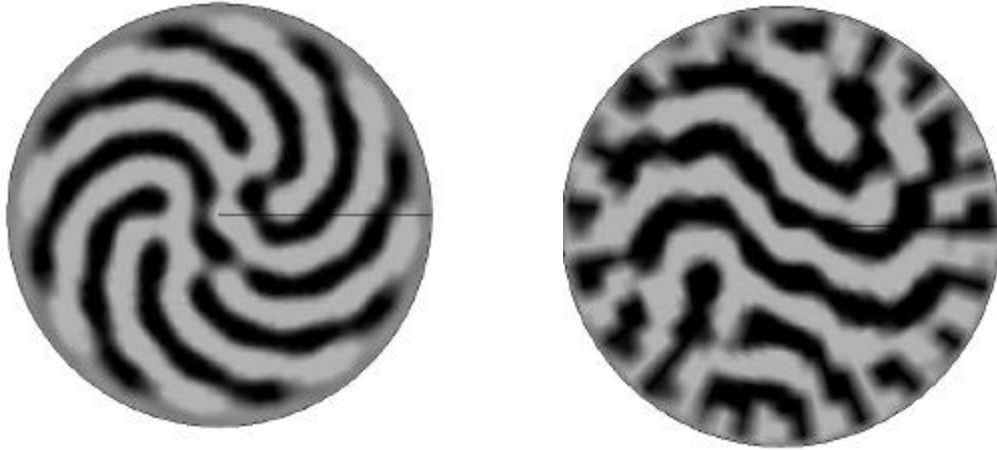


Figure 2: Iso-lines of temperature at mid-height in a cylinder $\Gamma=5$, $\Omega=60$ (conducting thermal sidewall condition). (a) $Ra=5600$, (b) $Ra=6000$, (c) $Ra=6800$, (d) $Ra=8000$.

At $Ra=5600$ the flow is steady near the axis of the cylinder and oscillatory in the bulk. At $Ra=6000$ the time behaviour is chaotic near the axis and oscillatory in the rest. The fast drifting wall mode was not detected. At $Ra=6800$, some spiral arms get connected at the cavity centre forming S-shaped structures. This contact brings about a quasiperiodic time behaviour and the signal presents a strong modulation. At higher Rayleigh number, $Ra=8000$ the time signal becomes chaotic within the entire cavity. The pattern is a quite organized structure with parallel S-shaped convective rolls which break and evolve into more chaotic patterns. The rolls wander and break forming some preferred orientation and the visualizations indicate the discontinuous nature of these orientation changes. These flow patterns thus show a behaviour which strongly reminiscent of the Küppers-Lortz instability.

Interaction of two counter rotating sidewall travelling waves - We studied the nonlinear flow in cylinders of annular cross section and thermally insulating sidewalls using different aspect ratio cavities. In all cases sidewall travelling waves appear at the onset. The waves appear attached to both the inner and the outer sidewall. The two waves rotate in opposite directions, as predicted by Hermann and Busse (1993) and by Kuo and Cross (1993). In an extended cavity, $L=5$, and $\Omega=60$ the two sidewall travelling waves were observed for $Ra=5600$ and rotate with the same phase velocity and in opposite senses. In the intermediate radial region the fluid is stationary. By increasing the Rayleigh number up to $Ra=9600$, a chaotic roll breaking regime was found, similar to the regime found in the conducting cylinder and the Küppers-Lortz regime. In a narrow cavity, $L=1.5$ at a rotating rate $\Omega=60$, the sidewall modes were observed at $Ra=5600$, interacting in the bulk region. Some pairing of the rolls which come in contact between the two cylindrical walls has also been observed. As the rolls travel in opposite directions, they move away and after some time the pairs break and a new pairing of rolls occurs.

REFERENCES

- [1] Goldstein, H.F., Knobloch, E., Mercader, I. & Net, M. 1993 Convection in a rotating cylinder. Part I. Linear theory for moderate Prandtl numbers, *J. Fluid Mech.* 248, 583
- [2] Herrmann, J. & Busse, F.H. 1993 Asymptotic theory of wall-attached convection in a rotating fluid layer *J. Fluid Mech.* 255, 183-194.
- [3] Kuo, E. Y. & Cross, M. C. 1993 Travelling-wave wall states in rotating Rayleigh-Benard convection. *Phys. Rev. E* 47 R2245-2248
- [4] Serre, E. & Pulicani, J. P. 2001 A 3D Pseudospectral method for convection in a rotating cylinder. *Comp. Fluids* 30 / 4, 491-519.