Oscillating Taylor-Couette Flow

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ABSTRACT

A flow transition in a coaxial rotating cylinder with only inner cylinder rotation was investigated using ultrasonic Doppler method for a wide range of Reynolds number up to 40 times critical Reynolds number (R_c). By analysing spatio-temporal velocity profiles, excitation functions of various transition modes such as wavy and modulated wavy modes were obtained using space-time Fourier expansion (2D-FFT) and Orthogonal Decomposition techniques, and a new fast azimuthal mode was identified and reported[1]. Based on the excitation functions, we reported that the energy supplied in the system by the inner wall is transferred cascading from the Couette flow→Taylor vortex flow (TVF)→Wavy (+Modulated wavy) flow (WVF & MWV)→Fast Azimuthal wavy mode (FAW)→Soft turbulence.

Besides the natural sequence of flow transition, it is known that each wavy mode might take an eigenstate of varied eigenvalues of wave number for a fixed geometry. For instance for Taylor Vortex flow, the wave number, namely a cell size, may vary for 0.8 to 1.2 times 2d (d : gap distance)[2]. In such a configuration, a spatial forcing is given to the system by setting a wavy wall on the stationary outer cylinder. On the other hand, there has been no attempt to make a forcing on the azimuthal wave. The eignenvalue of WVF (*m*) may vary from 4 to 7 and the appearance of any *m* state is a natural sequence. What determines this state is unknown. It is also not known to us if any attempt was made to force the *m*-state of the WVF or to control the transition sequence.

The objective of the present investigation is to see if there is any effect of a perturbation of oscillation of the inner cylinder on the selection of *m*-state, and if it is possible to control the flow transition or to inject energy directly to any desired wave mode by this means.

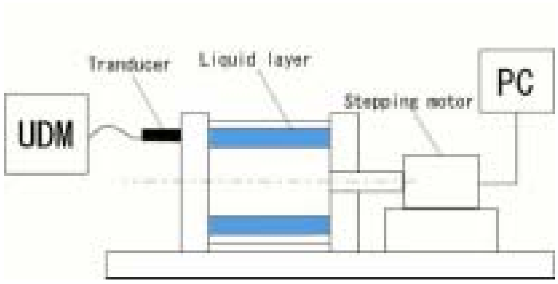

Fig. 1 Experimental set up.

The experimental configuration is the same (though newly made) as a previously used system (Fig.1); radius of inner cylinder (R_i) is 190 (mm) and outer cylinder (R_o) 210 (mm) and thus the radius ratio η=0.905. The height of the fluid layer (*h*) is 200 (mm) togive the aspect ratio Γ=20. The inner cylinder is fabricated with FRP to minimize its weight. Measurement is made by ultrasonic Doppler method (UDM) using UVP-X3 (Met-Flow S.A.). The ultrasonic basic frequency was 4MHz and the beam diameter is 4mm∅. The transducer was set at the outside of the gap with its beam center located at 2 mm from the outer wall. Liquid is a water-glycerole mixture. Reynolds number is defined as $R=d\Omega R_i/\nu$ (Ω a rotational speed, νkinematic viscosity). A perturbation of oscillation is given by changing the Reynolds number as

$$R(t) = R_w + \Delta R = R_w\,(1+A\sin(2\pi f_0 t))$$

where f_0 is a frequency observed in the power spectrum for the basic WVF and A is an oscillation amplitude to be varied.

The experimental procedure is as following; Start up the system for a fixed Reynolds number (R_w=3.0) for WVF mode and measured velocity profiles. Compute a space-averaged power spectrum

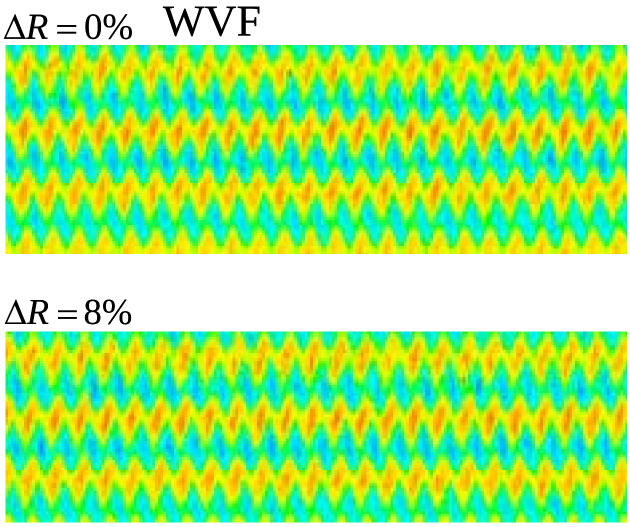

Fig. 2 Color density plot of measured velocity profiles; a coordinate is space (59.2-133.2 mm) and an abscissa is time (0.-65 sec).

to find the oscillation frequency f_0. Start oscillation of the inner cylinder for a range of its amplitude A. We changed this amplitude from 1% to 15%.

UDM measurement covers the spatial range from 0. mm to 188 mm with time resolution of 130 msec. This covers ca. 4 roll pairs and ca. 21 rotations of the inner cylinder. Fig.2 is an example of measured spatio-temporal velocity profile, represented as color density plot.

A data set (128 spatial points and 1024 profiles) was first treated to obtain a space dependent power spectrum and then averaged over the space. As seen in Fig.3, most of the spectra are clean due to spatial averaging, showing only a few peak lines. In Fig.3a, it is readily identified that the lowest peak (f_1) corresponds to the fundamental WVF mode and the rest (f_2 and f_3) are its higher harmonics. In Fig.3b, however, the frequency of the fundamental mode is slightly different from that of Fig.3a. Such a frequency shift might happen due to a change of *m*-state (probably by one), but it does not always occur. We repeated the experiment but found no rule when and how it occurs. It should also be noted that we could not control to set the *m*-state. Occurence of any eigenstate was random.

Fig.4 shows a sequence of power spectra with respect to a change of oscillation amplitude, A, up to 15%. It is observed that the power spectra show little change in their feature. For a large amplitude than 10%, a peak appears at slightly higher frequency from the harmonic peaks, but the power is very small. Such pictures were obtained for both ascending and descending changes of the amplitude and no hysterisis was observed. As the range of variation of Reynolds number for the largest amplitude oscillation stays within the range where only the WVF mode prevails. Even at the maximum Reynolds number occuring as $R_w(1+A)$ is still lower than the critical Reynolds number for the onset of MWV (R_M=5.59). It is therefore not surprising that all power spectra show similar feature, except for the appearance of a small peak (f_4) beside the 2. harmonics (f_2).

Total power of each peak was estimated from power spectra and plotted with respect to a change of the amplitude of oscillation (Fig.5). This sort of plot is an excitation function, which indicates how strongly each mode is excited with respect to a control parameter.

For the main modes (f_1 to f_3) there was observed a peculiar variation of the power. The main mode (f_1) keeps its power only up to A≈2% and then

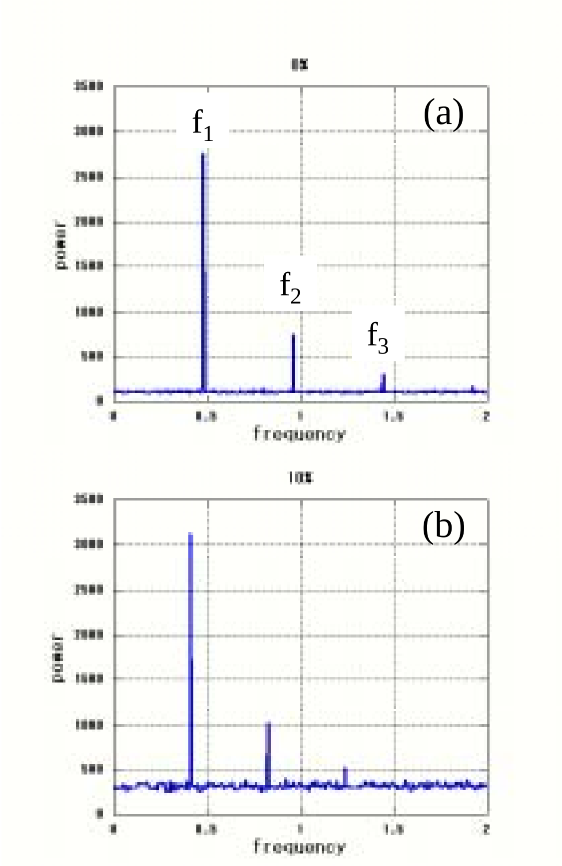

Fig. 3 Power spectra for (a) WVF steady oscillation A=0, (b) with 5%.

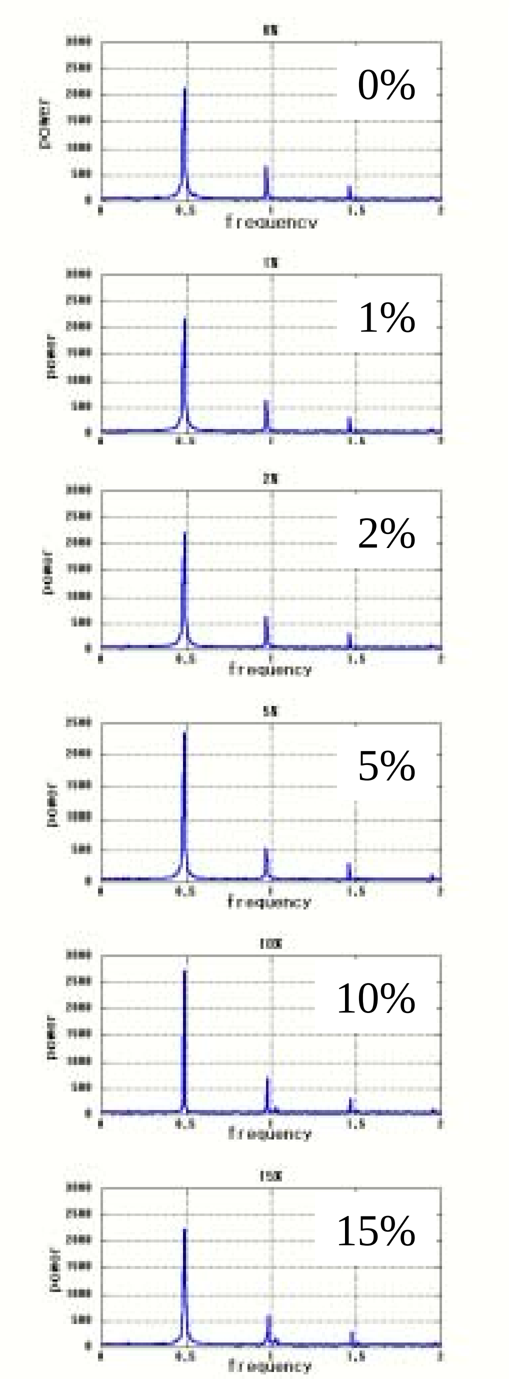

Fig. 4 A sequence of averaged power spectra.

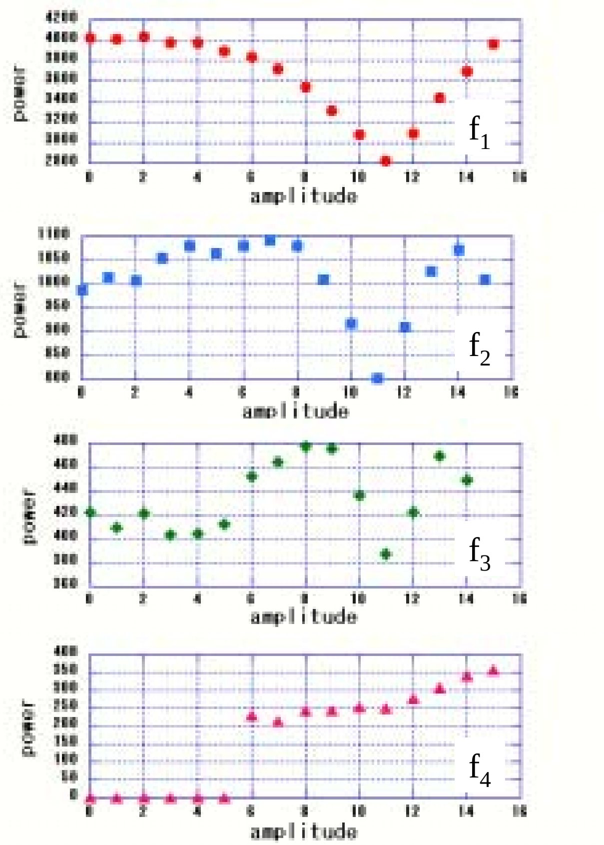

Fig. 5 Excitation function of largest eigen modes; a variation of the peak with respect to the oscillation amplitude.

gradually decreases, to show the minimum at A=11%. The second harmonics (f_2) shows a gradual increase from the beginning and a fairly sharp dip at A=11%. The third harmonics (f_3) keeps its power upto A=5% followed by an increase, and then it also shows a dip at A=11%. The dip of the power occurs at the same oscillation amplitude, 11% in the shown example, for three harmonic components. Repeating the experiments, we found that the dip occurs all the cases studied but at slightly different oscillation amplitude; 8~11%.

These three components occupies most of the total energy, and it is not known to where those energy that corresponds to a decrease of energy occuring at A=11% is transferred as long as the axial velocity component is concerned. Although two higher harmonics show an increase of their power for the lower amplitude, the amount is not sufficient to compensate the decrease of the main mode.The only possible reason would be the energy exchange between the axial compoent and radial/azimuthal component.

Fig.6 shows a spatial distribution of the power of the main peak, namely a space dependent power spectrum for A=0, 8 and 11%. It shows a weak tendency that this mode prevails with a larger size at inflow boundary of the cell pairs than that for the outflow boundary. This tendency is similar to any higher frequency components occuring in the MWV as previously reported[3]. It is however seen that the peak becomes sharper for the larger oscillation amplitude. It indicates that the energy is more concentrated on that frequency and that the effect of temporal forcing is significant.

The mode corresponding to a very small peak (f_4) near f_2 appears at A=6% and its growth is very weak. It does not show any similarity in the variation of power with respect to the oscillation amplitude but no dip of the power is observed. It is therefore concluded that this weak mode is a completely different mode from the WVF mode.

In conclusion, eigen state (*m*-state) of WVF cannot be controlled by oscillation of the inner cylinder. It is a natural occurece which state to be taken, and cannot be selected externally. This means that the degeneracy of different *m*-state could not be resolved by this method.

At least we might say that the energy cascading in this system is not broken as initially thought. The direct energy injection to the higher mode is not realized. This result suggests us that the energy of axial velocity component would be transferred to radial or azimuthal velocity components.

There are two points to be mentioned as an effect of oscillation of the inner cylinder. The clear result is that the variation of the power of the modes of WVF (main peak and its higher harmonics) shows a dip at 11% of the oscillation aimplitude. This amplitude is, however, not definite and sactteres from run to run.

By increasing the oscillation amplitude, the peak in the power spectrum becomes sharper. This indicates that the WVF mode is considerably enhanced by the external temporal forcing.

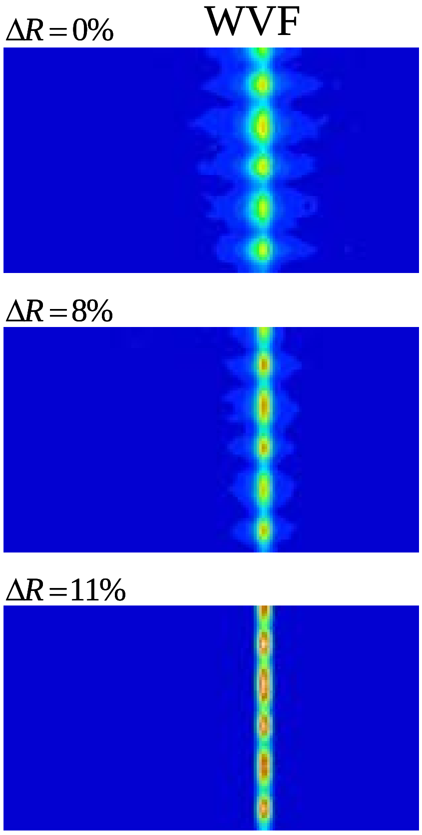

Fig. 4 Space dependent power spectra; a coordinate is space (59.2-118.4 mm) and an abscissa is frequency (0.3-0.6 Hz).

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2 R.C. DiPrima and H.L. Swinney, Instability and transition in flow between concentric rotating cylinders, in Hydrodynamic Instabilities and the Transition to Turbulence, H.L. Swinney and J.P. Gollub, Eds., Springer-Verlag, Berlin, 1985

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