Heteroclinic cycles and Kelvin-Helmholtz instability in the flow between exactly counter-rotating disks

Caroline Nore[1]**, Laurette S. Tuckerman**[2]**, Olivier Daube, Shihe Xin**

Laboratoire d'Informatique pour la Mécanique et les Sciences de l'Ingénieur (LIMSI-CNRS)
B.P. 133, 91403 Orsay, France
[1]e-mail: nore@limsi.fr - Web page: http://www.limsi.fr/Individu/nore/
[2]e-mail: laurette@limsi.fr - Web page: http://www.limsi.fr/Individu/laurette/

ABSTRACT

The flow in a cylinder engendered by the differential rotation of the upper and lower bounding disks is called von Kármán flow. Like Taylor-Couette flow, this flow has three non-dimensional parameters, which can be taken to be an aspect ratio, an angular velocity ratio, and a Reynolds number. Unlike Taylor-Couette flow, however, only a few parameter combinations have been studied, e.g. numerically by Gelfgat et al. [1] and Lopez et al. [2] and experimentally by Schoveiler et al. [3] and Gauthier et al. [4]; the three-dimensional patterns and transitions remain unexplored for most parameter values. We have carried out a detailed study of the case in which the two disks rotate in equal and opposite directions. This is the only configuration of von Kármán flow which has $O(2)$ symmetry: the configuration remains invariant under rotations S_θ in θ and also under combined reflection R_π in θ and z (equivalent to rotation by π about the x axis). The $O(2)$ symmetry has important consequences, especially when modes with azimuthal wavenumbers $m = 1$ and $m = 2$ compete, which is the case when the height-to-radius ratio Γ of the cylinder is close to 2, the geometry which we have studied. In this case, the normal form analysis by Armbruster et al. [5] predicts the existence not only of stationary states with wavenumbers $m = 1$ and $m = 2$, but also of traveling and modulating waves, and of heteroclinic cycles. A schematic bifurcation diagram is shown in figure 1. The eigenvectors leading to the $m = 1$ and $m = 2$ steady states are shown in figure 2.

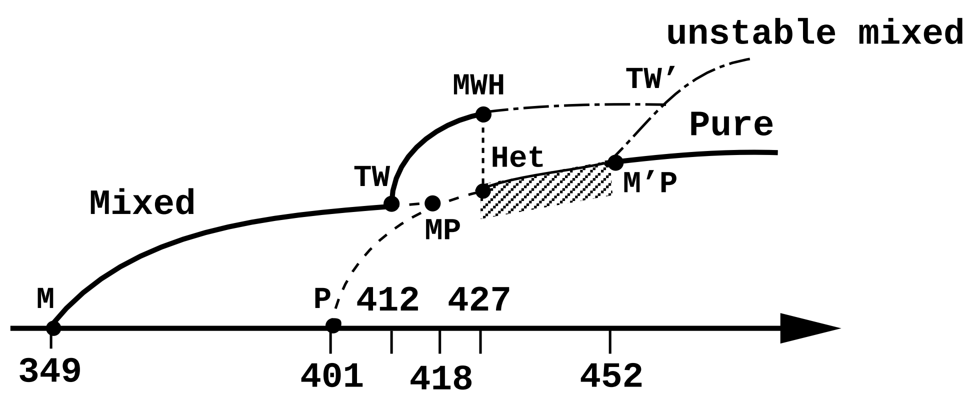

Figure 1: Schematic bifurcation diagram as a function of Re. Stable solutions are indicated by solid lines, unstable ones by dashed lines and shaded lines denote attracting heteroclinic cycles. Dot-dashed lines indicate branches which we have not computed. Thresholds are indicated by dots: $Re_{\rm M} = 349$, $Re_{\rm P} \simeq 401$, $Re_{\rm TW} \simeq 412$, $Re_{\rm MP} \simeq 418$, $Re_{\rm MWH} = 427.3$, $Re_{\rm Het} = 427.4$, $Re_{\rm M'P} \simeq 452$.

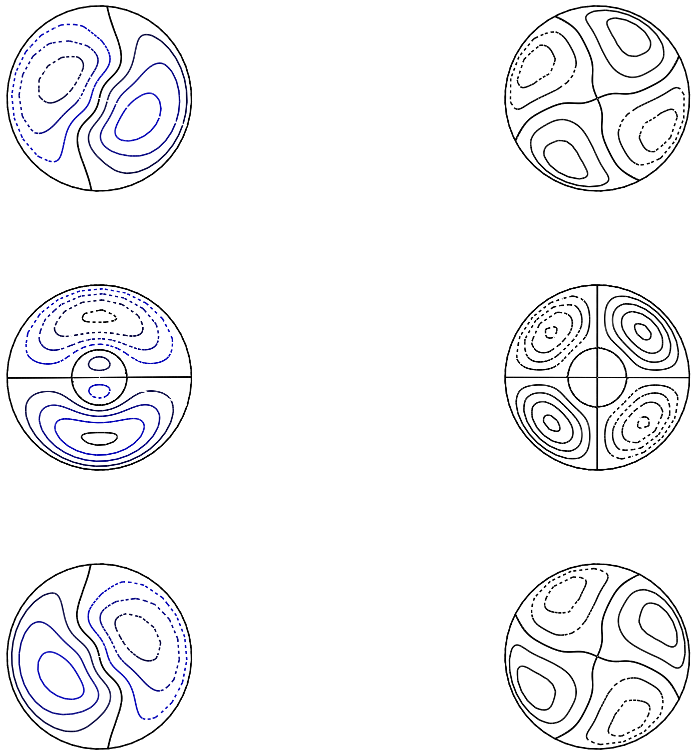

Figure 2: Vertical velocity contours at $z = \Gamma/6$ (top), $z = 0$ (middle), and $z = -\Gamma/6$ (bottom) of the eigenvectors (a) $m = 1$ at $Re = 355$ and (b) $m = 2$ at $Re = 410$.

The most exotic consequence of the 1:2 mode interaction is the existence of heteroclinic cycles over a range of Reynolds number and which connect two $m = 2$ states differing by rotation by $\pi/2$. The cycles which we observe numerically have a long but finite period, and hence are termed nearly heteroclinic. Figure 3 illustrates. the sequence of flows occurring during such a cycle. The vertical velocity timeseries plotted in figure 4 show that the cycle consists of two long plateaus, punctuated by two rapid changes. We have also also observed nearly heteroclinic cycles containing four plateaus, but not the chaotic cycles seen by Mercader et al. [6]. The circumstances under which these different types of cycles occur are at present unknown.

We have proposed a physical mechanism for the primary instabilities towards the $m = 1$ and $m = 2$ states. The axisymmetric basic state between two counter-rotating disks contains a narrow shear layer at mid-height over which the azimuthal velocity varies greatly and which dominates the dynamics [2]. We may hypothesize that the instability is a generalization to cylindrical profiles $v_\theta(z)$ of the Kelvin-Helmholtz instability, as suggested by the resemblance between the vortices in the eigenvectors of figure 2 to the classic cat's-eyes form. A quantitative analysis agrees fairly well with predictions of critical wavenumbers and Reynolds numbers for the Kelvin-Helmholtz instability.

Finally, we have calculated the linear stability of counter-rotating von Kármán flow for aspect ratios Γ ranging between 0.5 and 3, and observed that the critical wavenumber obeys $m_C \sim 1 + [2/\Gamma]$; see figure 5. This concords with the idea that the instability is localized in the shear layer, which itself occupies a constant proportion of the height.

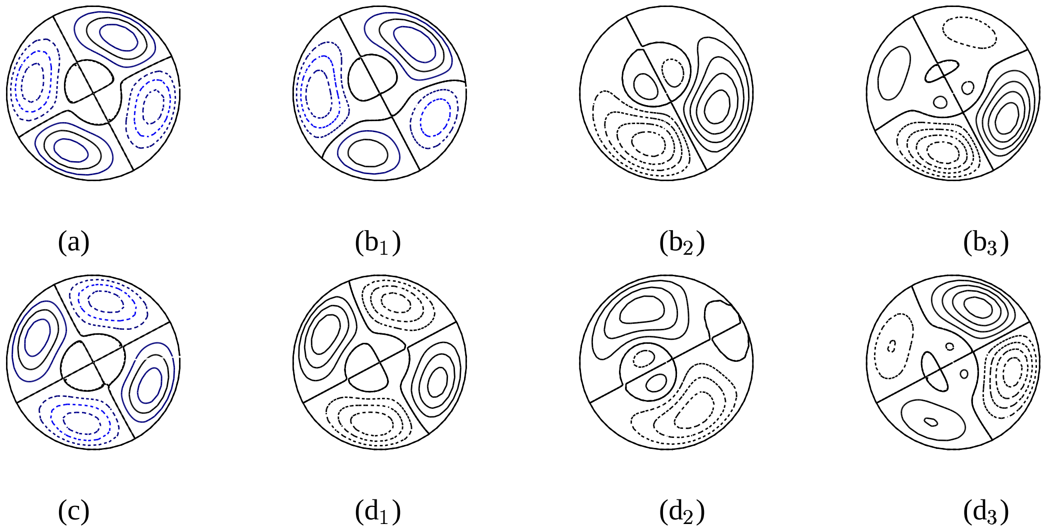

Figure 3: Vertical velocity contours at $z = 0$ for a heteroclinic cycle at $Re = 435$. This cycle connects the two $m = 2$ states (a) and (c). Intermediate states, dominated by $m = 1$ and other odd Fourier components, are shown in (b$_1$, b$_2$, b$_3$) and (d$_1$, d$_2$, d$_3$).

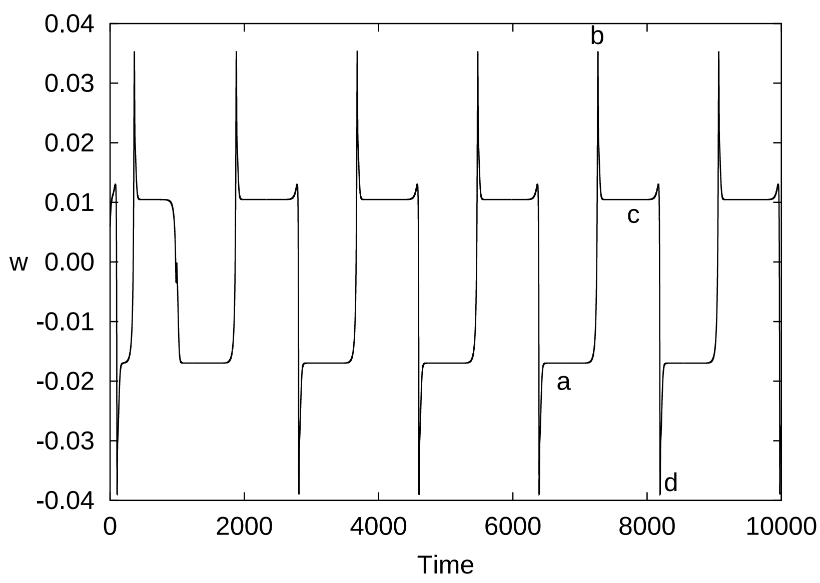

Figure 4: Velocity $w(1/2, 0, 0)$ during a heteroclinic cycle at $Re = 435$.

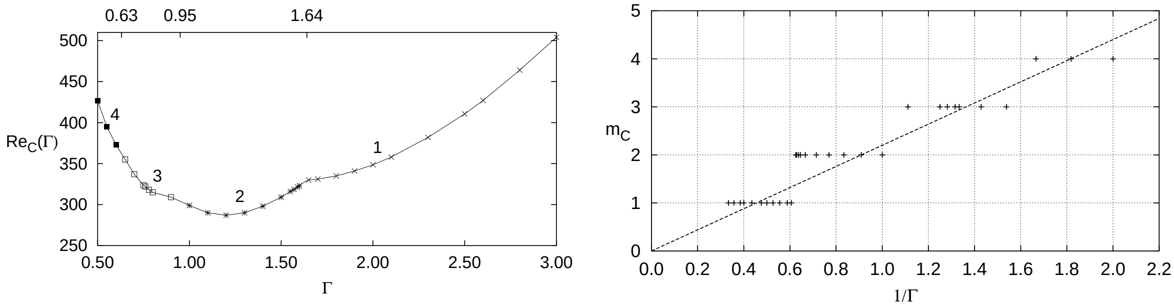

Figure 5: Left: critical Reynolds number Re_C as a function of Γ realised for different critical wavenumbers: $m = 1$ ($\times$), $m = 2$ ($*$), $m = 3$ ($\square$), $m = 4$ ($\blacksquare$). Right: critical wavenumber m_C ($+$) as a function of $1/\Gamma$, with the indicative dashed line $2.2/\Gamma$.

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