Rotating Shear Flows and Subcritical Turbulence

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ABSTRACT

The present investigation has two major objectives. First, the relation between rotating plane Couette flows and Taylor-Couette flows is clarified by reexpressing the Navier-Stokes equation of Taylor-Couette flows in the rotating frame. In the process, it appears that the dimensionless numbers that best characterize Taylor-Couette flows from the point of view of the flow dynamics are the Reynolds number, the rotation number ($Ro = 2\Omega\Delta R/(R\Delta\Omega)$), and an appropriately defined curvature number R_c, which reduces to the relative gap width ($R_c = \Delta R/R$). Taylor-Couette flows reduce to rotating plane Couette flows when $R_c \rightarrow 0$ at finite R_o. This limit implies that there must be a connection between Rayleigh's stability criterion and the stability limit of rotating flows [1,2]. Indeed, it is shown that the (de)stabilizing effect of the angular momentum radial distribution can be reinterpreted in terms of the (de)stabilizing effect of the Coriolis force in the rotating frame. This allows us to reexpress Rayleigh's criterion in terms of the flow rotation number, so that the linear stability criteria of rotating plane Couette flows and Taylor-Couette flows become formally identical [3,4]. More precisely, they are both linearly stable when $-1 < S < 0$ where $S = 2\Omega/(dU/dy)$ for rotating plane Couette flows, and $S = 2\Omega/(rd\Omega/dr)$ for Taylor-Couette flows.

The second objective is to understand the transport properties of subcritical turbulence in rotating flows. By analyzing all (scant) data on subcritical turbulence in rotating plane Couette flows [5] and Taylor-Couette flows [6,7,8,9], the dependence of the minimum Reynolds number for the onset of turbulence in subcritical Taylor-Couette flows with respect to the rotation number and the curvature number is disentangled and characterized. The connection between Taylor-Couette and rotating plane Couette flows is confirmed by the consistency of the data in the appropriate parameter limit, and used in this analysis. The data imply that the minimum Reynolds number increases linearly with the rotation number, and quadratically with the curvature number [4]. A phenomenological analysis is presented, which provides an explanation for this dependence on gap width in the old experiments of Taylor with the inner cylinder at rest, a fact which has apparently not been properly explained before [10]. This phenomenological analysis allows us to relate the minimal Reynolds number of onset of turbulence in subcritical flows to the turbulent momentum transport efficiency, a crucial feature from a practical point of view [4,10]. It is shown that, as a consequence, the Coriolis force reduces the efficiency of turbulent transport with respect to nonrotating flows, as observed in numerical simulations [10,11,12]. Implications for subcritical turbulent transport in astrophysical accretion disks are discussed; the role of the disk scale height is pointed out.

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