Travelling waves and transition to turbulence in pipe flow

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ABSTRACT

Hagen-Poiseuille flow through a pipe of circular cross section belongs to the class of shear flows that does not become linearly unstable. The situation is similar to Taylor-Couette flow with the inner cylinder at rest as well as to Taylor-Couette in the limit of large radii where the system approaches plane Couette flow [3]. In these cases the transition to turbulence is not related to series of symmetry-breaking linear instabilities but rather with the formation of nonlinear 3-d states that are not connected to the laminar profile and that seem to form a chaotic saddle.

As indicators for this process we present exact coherent states to pipe flow, we discuss the sensitive dependence on initial conditions quantified by the Lyapunov exponent of turbulent trajectories, and the exponential distribution of life times of the turbulent state. All our findings are consistent with the formation of a strange saddle (repellor) in phase space.

For the numerical solution of the full Navier-Stokes equation we use a Fourier-Legendre collocation method in cylindrical coordinates, with Lagrange multipliers to account for no-slip boundary conditions at the wall and the constraints that the flow field is solenoidal, analytical and regular at the centerline. The code was verified by reproducing literature values for the linearized problem, for the nonlinear dynamics of optimal modes and for the statistical properties of fully developed turbulent flow up to Reynolds numbers of 5000.

A family of three-dimensional **travelling waves** for flow through a straight pipe of circular cross section has been identified (Fig. 1,2) [1]. They were found by a Newton-Raphson method which was implemented with a spatial resolution of 21 modes in azimuthal and downstream direction and 44 Legendre polynomials radially. This gives us about 8700 dynamically active velocity coefficients.

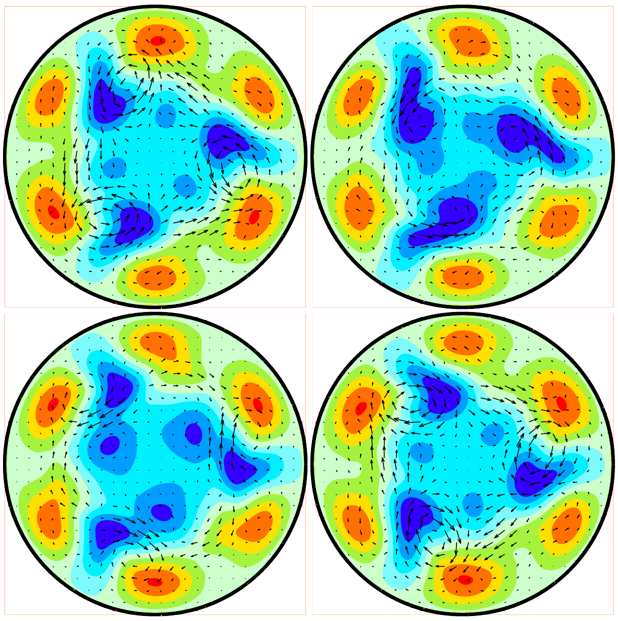

Figure 1: Travelling wave with threefold azimuthal symmetry at the bifurcation at $Re = 1250$. The frames are cross sections at different downstream positions separated by $\Delta z = L_z/6$. Only half a period is shown: the last frame is the same as the first one up to a reflection at the horizontal diameter ($\phi \to -\phi, u_\phi \to -u_\phi$). Velocity components in the plane are indicated by arrows, the downstream component by color coding: velocities faster than the parabolic profile are shown in green/yellow/red, slower ones in blue.

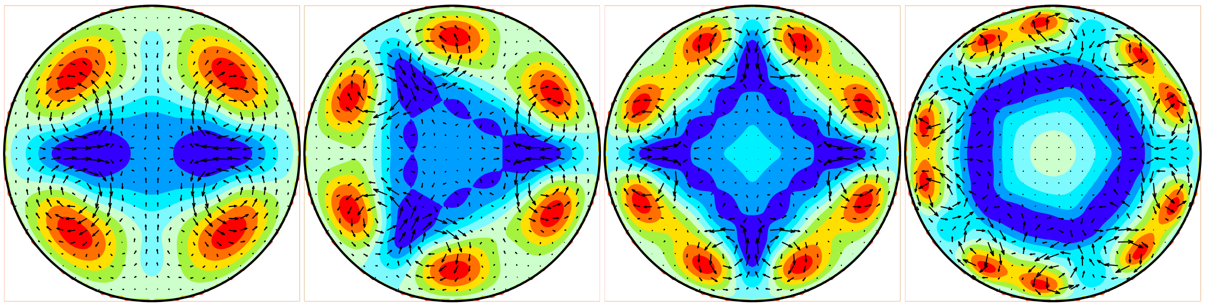

Figure 2: Travelling waves with symmetries C_n with $n = 2, 3, 4,$ and 5 (invariance under rotation around the pipe axis by an angle $2\pi/n$). In order to highlight the topology of the states all states are averaged in downstream direction. The representation of the velocity field by vectors (in-plane motion) and color (downstream component) is as in Fig. 1. The absolute scale for the velocity fields is given in Table 1.

They originate in saddle-node bifurcations at Reynolds numbers as low as 1250, where Re is based on the mean downstream velocity and the pipe diameter. All states are immediately linearly unstable at the bifurcation.

The travelling waves are dominated by pairs of downstream vortices and streaks (Fig. 1,2). The high speed streaks near the wall move much less than the low speed streaks closer to the center of the pipe. The dominating structures are streamwise streaks and streamwise vortices that closely resemble coherent states in other shear flows like the wavy-vortex flow in Taylor-Couette or plane Couette flow. Some selected properties of the waves at bifurcation are given in Table 1. The dynamical significance of the exact coherent states is that they provide a skeleton for the formation of a chaotic saddle that can explain the intermittent transition to turbulence and the sensitive dependence on initial conditions in this shear flow [1,2].

For dynamical **lifetime experiments** a spatial resolution of 33 modes in azimuthal direction, 29 modes in downstream direction and 50 Legendre polynomials radially has been used. For an accurate simulation of turbulent dynamics at transitional Reynolds numbers the streamwise periodicity L is set to $10R$ which is about the minimal value needed to justify periodic boundary conditions, i.e., for velocity fluctuations to be uncorrelated at a streamwise separation of half the pipe length. As in the experiments of Darbyshire & Mullin [4] we keep the volume flux constant. We extended our numerical investigations to times of 2000 or more (unit of time: mean streamwise velocity/radius), far exceeding the values accessible in the longest currently available laboratory experiment. As initial conditions we used a high amplitude uncorrelated superposition of spectral modes. The spatial structure is so rich and the amplitude so high that the probability to trigger turbulent dynamics is maximal for a wide range of Reynolds numbers.

Various conclusions can be drawn from our lifetime experiments: We find a transitional Reynolds number somewhat below $Re = 2000$. The minimum amplitude to trigger a long living turbulent dynamics decreases with Reynolds number. We observe strong sensitivity on initial conditions as well as on parameters which results in large fluctuations in turbulent life times (Fig. 3) [2]. The regions of quickly decaying and long-living trajectories are separated by complicated, fuzzy stability borders. The results are in agreement with experiments by Darbyshire & Mullin [4].

In order to quantify the sensitive dependence on initial conditions the largest Lyapunov exponent has

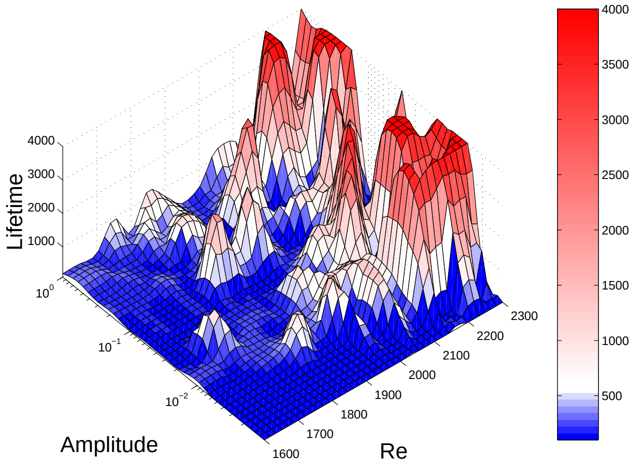

Figure 3: Lifetime of turbulent trajectories as function of Re and initial disturbance amplitude. The color-coding separates runs that would have decayed within usual experimentally accessible lifetimes and those that would have appeared as sustained.

been calculated along ensembles of turbulent trajectories. Its typical value is about 6.5×10^{-2} at transitional Reynolds numbers and it slowly increases with Re. This corresponds to an amplification factor of the order of 10^6 over 200 time units which is a typical time scale for a nonlinear regeneration cycle. The short time Lyapunov exponents are strongly correlated with the large energy fluctuations in the system. When new large scale structures are generated the energy grows strongly and the Lyapunov exponent increases whereas at the end of a nonlinear regeneration cycle the energy goes down and it decreases as well.

The strong sensitivity of lifetimes on initial conditions and on parameters calls for a statistical description of the transition process. Therefore, the distribution of turbulent lifetimes has been calculated and it is shown in Fig. 4.

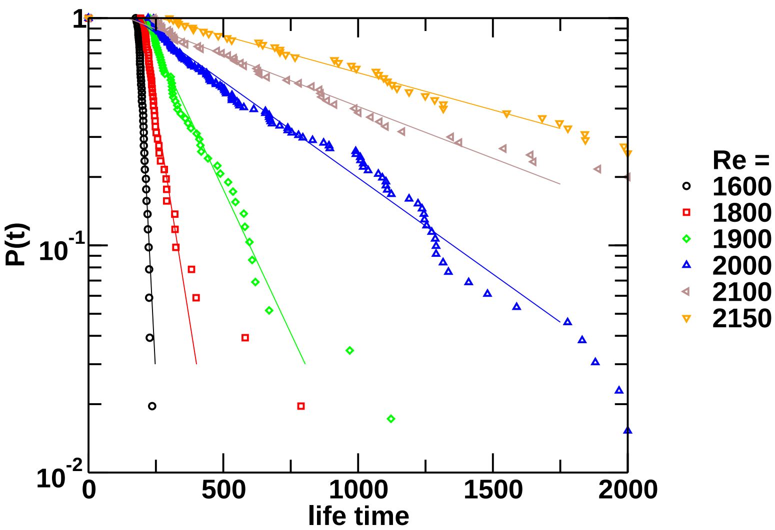

Figure 4: The probability for a single trajectory to still be turbulent after a certain integration time. Between 50 and 100 trajectories have been integrated per Reynolds number.

Although the fluctuations are large an exponential distribution can be identified within the statistical errors. This can be interpreted as a constant rate of escape from the turbulent state which is a major characteristics of a chaotic repellor.

Increasing the Reynolds number up to $\approx$ 2200 the median of the turbulent life times as well as the fluctuations increase rapidly until the median reaches the cut-off life time of 2000. It might even be the case that the escape routes from the repellor are closed and it is turned into a strange attractor.

symmetry	C_2	C_3	C_4
Re_c	1350	1250	1590
L_c/R	4.19	2.58	2.51
v/U	1.43	1.29	1.17
n_u	2	1	4
u_s/U	0.38	0.35	0.34
u_p/U	0.035	0.046	0.045

Table 1: Selected properties of travelling waves at the saddle-node bifurcation. Given is the critical Reynolds number Re_c at the optimal wave length L_c, the phase velocity v and the number of unstable dimensions n_u. u_s is the maximum deviation of the streamwise velocity from the laminar flow, u_p is the maximum in-plane velocity component.

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