A Computational Study of Unsteady Wake of a Bluff Body in Density-Stratified Flow

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ABSTRACT

Stratified flow past a three-dimensional obstacle such as a sphere has been a long-lasting subject of geophysical, environmental and engineering fluid dynamics. In order to investigate the effect of the stratification on the near wake, in particular, the unsteady vortex formation behind a sphere, numerical simulations of stratified flows past a sphere are conducted. The time-dependent Navier-Stokes equations are solved using a three-dimensional finite element method and a modified explicit time integration scheme. Laminar flow regime is considered, and linear stratification of density is assumed under Boussinesq approximation. The computed results include the characteristics of the near wake and the unsteady vortex shedding. With a strong stratification, the separation on the sphere is suppressed and the wake structure behind the sphere becomes planar, resembling that behind a vertical cylinder.

INTRODUCTION

Wake downstream of a sphere in the stratified flow has been of long-standing interest in fluid dynamics because of its similarity to geophysical flow over topographical terrains and more recently, concerns about the wake left behind a body moving through the ocean thermocline. Among unique and interesting characteristics of the stratified flow past obstacles are the tendency of the flow to be confined to horizontal planes and the formation of planar vortex streets when the stratification is strong [1]. In laboratory experiments, Debler & Fitzgerald [2] observed the suppression of the separation behind a sphere under strong stratification and Brighton [3] conducted an experimental study of the stratified flow past various obstacles, including hemisphere, in the boundary layer. The results showed that the flow tends confined to horizontal planes and, under the influence of strong stratification, evolves to the planar vortex streets which resembles von Karman vortex.

For a sphere in the stratified flow, experimental observations were made recently by Lin et al [4] and Chomaz et al [5] for a wide range of both Reynolds and Froude number, Re and Fr where $Re = U_O D/\nu$ and $Fr = U_O/ND$;

the symbols U_O and D denote the oncoming velocity and the sphere diameter, respectively; N denotes the buoyancy frequency of the density-stratification. When the stratification is strong ($Fr < 0.4$), the vertical displacement of the flow is suppressed, and the wake remains nearly two-dimensional on the horizontal plane. For very strong stratification at $Fr < 0.25$, development from stationary, attached eddy to unsteady vortex shedding was observed. With less strong stratification (Fr close to 0.4), the increase in the amplitude of the lee waves on the vertical plane is eminent, and the separation line on the sphere becomes bow-tie shape. When Fr reaches 0.4, the separation on the sphere surface is nearly suppressed, and the amplitude of the lee wave is maximized.

Hanazaki [6] presented the computational results of the stratified flow past a sphere at $Re = 200$. Inspite of unsteady formulation, the simulated wake remains stationary for all Froude numbers considered ($0.125 < Fr < 100$), whereas many experimental studies discussed above have observed the development of planar vortex shedding on the horizontal plane at low Fr.

The current work attempts to numerically solve three-dimensional Navier-Stokes equations for the flow in a time-dependent manner and analyze the dynamics

and structure of the near wake behind a sphere submerged in density-stratified flow.

FORMULATIONS

First, considering a viscous, incompressible flow with a linear density-stratification in the vertical direction (x_3 or z-axis in the present study), the governing equations are given as

$$u_{j,j} = 0 \qquad (1)$$

$$\rho_t \left(u_{i,t} + u_j u_{i,j} \right) = -p_{,t} - \rho_t g \delta_{i3} \atop + \mu \left(u_{i,j} + u_{j,i} \right)_{,j} \qquad (2)$$

$$\rho_{t,t} + u_j \rho_{t,j} = 0 \qquad (3)$$

where subscript t represents a total quantity and diffusion term is neglected in Eq (3). Introducing Boussinesq approximation, it is assumed that the density perturbation is small compared to reference density and affects only on the bouyant term in the momentum equations; hence Eq (2) and (3) becomes

$$u_{i,t} + u_j u_{i,j} = -p_{,i} - \delta_{i3} \, \rho / \mathrm{Fr}^2 + \tau_{ij,j} \quad (4)$$

$$\rho_{,t} + u_j \rho_{,j} = u_3 \qquad (5)$$

In the present study, a finite element method is applied to Eq (1), (4) and (5), leading to

$$\int_\Omega \psi u_{j,j} d\Omega = 0$$

$$\int_\Omega \left(\phi u_{i,t} + \phi u_j u_{i,j} + \phi_{,j} \tau_{ij} - \phi_{,i} p \right) d\Omega$$

$$= -\tfrac{\delta_{i3}}{\mathrm{Fr}^2} \int_\Omega \phi \rho d\Omega + \int_\Gamma \phi \sigma_{ij} n_j d\Gamma$$

$$\int_\Omega \left(\phi \rho_{,t} + \phi u_j \rho_{,j} \right) d\Omega = \int_\Omega \phi u_3 d\Omega$$

where Ω is an arbitrary finite element of which the boundary is denoted by Γ. ϕ and ψ are the weight functions for velocity, density and pressure, respectively. A Petrov-Galerkin type formulation using streamline upwind method [7] is employed for numerical stability. The unsteady computations are carried out by a modified explicit method for velocity whereas Crank-Nicolson scheme is employed for density transport due to the absence of the density diffusion. Pressure poisson equation is separately solved at each time step.

RESULTS

Figures 1 and 2 show the computational domain of cylindrical shape and mesh distribution close to the sphere, respectively. In the present study, time-dependent computations are carried out for $Re=200$ and $0.02 \le Fr \le 100$, in which only linear stratification of density is considered.
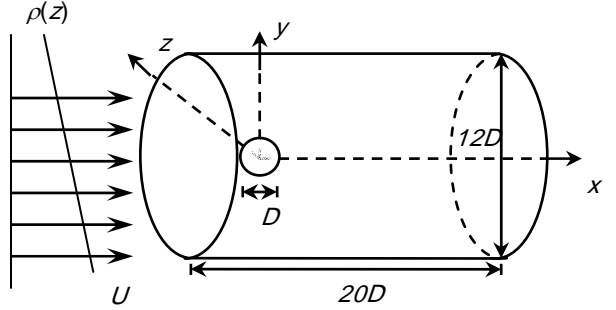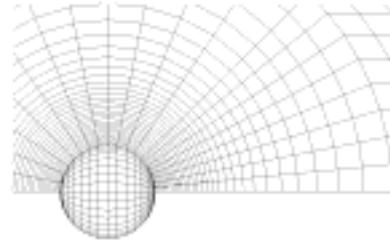

Figure 1. Schematic diagram of computational domain

Figure. 2 Mesh distribution near sphere

Figure 2 shows streamlines on vertical and horizontal planes for $Fr =10$, 1, 0.5, and 0.2 at $Re = 200$. An axisymmetric shape of the near wake is observed for $Fr =10$ (a). As stratification increases to $Fr =1$ (b), the separation points on the vertical plane move closer to the rear stagnation point. With the increase of the stratification, the wave length of the internal wave decreases and the trough move closer to the back of the sphere. As stratification further increases to $Fr =0.5$ (c), the recirculation zone in the near wake collpases completely. Lin et al [4] observed the occurence of the unsteady vortex shedding at $Fr =0.25$ for $Re =200$; however, the present computation remains stationary up to $Fr =0.2$ (d).

When Fr becomes smaller than 0.2 in the present study, unsteady vortex shedding occurs on the horizontal planes. Figure 4 shows the velocity vector fields for $Fr =0.1$. The unsteadiness computed in the present study

agrees well with the experimental results [4,5] and the computed Strouhal number of the shedding for Fr =0.11-0.05 is about 0.196 which also agrees well with the measured data of 0.2 [4,5].
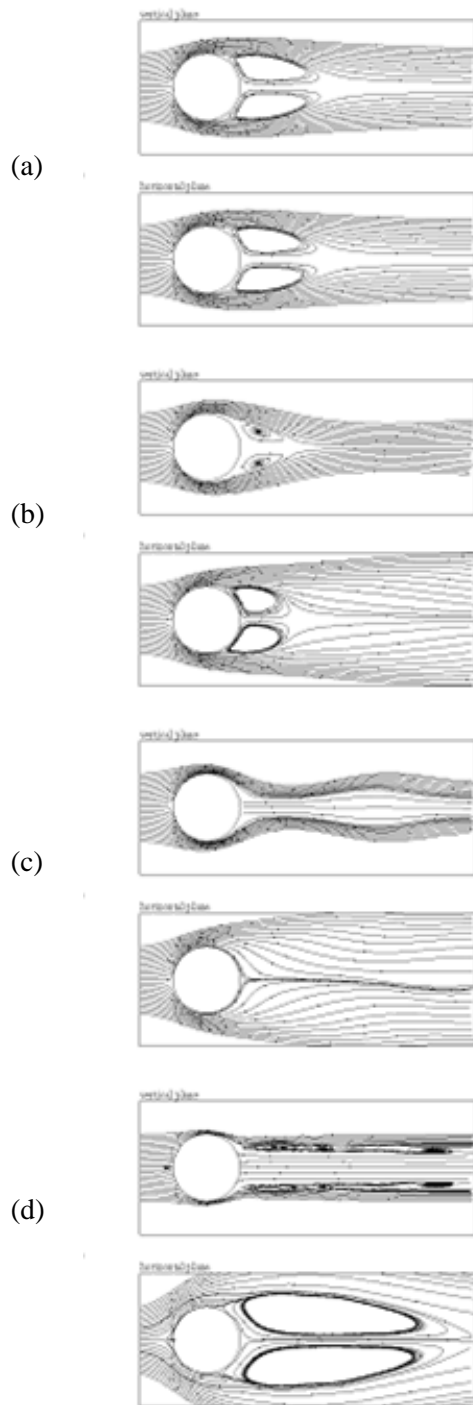

(a)

(b)

(c)

(d)

Figure 3. Side views on vertical plane ($x-z$) and plan views on horizontal plane ($x-y$) of streamlines for Fr= 10 (a), 1 (b), 0.5 (c) and 0.2 (d) at Re= 200
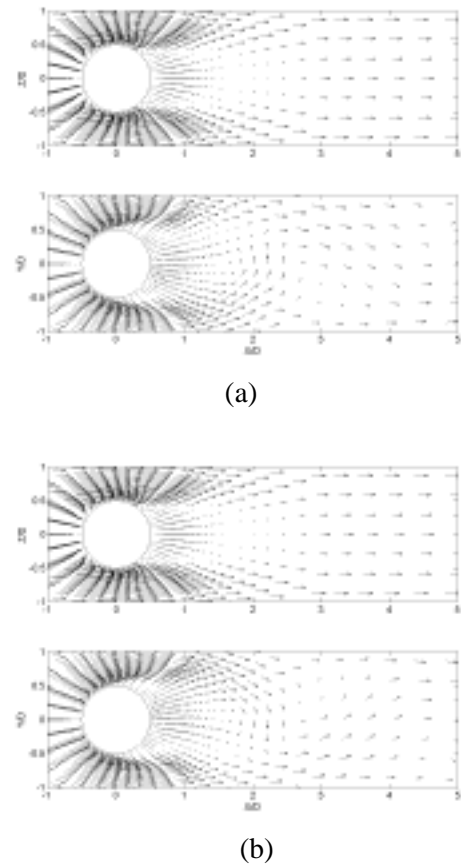

(a)

(b)

Figure 4. Side views on vertical plane ($x-z$) and plan views on horizontal plane ($x-y$) of velocity vectors for $Fr = 0.1$ and $Re = 200$ at t=0 (a) and $t=T/2$ (b) where T is a vortex shedding frequency

CONCLUDING REMARKS

- A finite element analysis of the stratified flow past a sphere is carried out for varied stratification at Re =200 and the experimental observations [4,5] are well simulated.

- With the increase of stratification, the separation occurs further close to the rear stagnation point and the recirculation collapses at Fr=0.5.

- When Fr<0.2, the near wake is almost planar on the horizontal planes and unsteady vortex shedding occurs in the present computation. The unsteadiness has been observed only in the experiments not in previous computational study [6].

ACKNOWLEDGEMENTS

Program of the Korea Science & Engineering Foundation.

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