# Natural Convection in rotating Spherical gap under the central Force Field

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## ABSTRACT

Convection in spherical shells under the influence of a radial force field is an important problem in classical convection theory. It is difficult to reproduce in a terrestrial laboratory though, because there gravity is everywhere downwards rather than radially inwards. It turns out that one can produce a radial force field, by applying a voltage difference between the inner and outer spheres. The combination of the electric field and the temperature-dependence of the fluid's dielectric coefficient then produces an $r^{-5}$ central force field. The convection flow is investigated in the cases without and with rotation as a function of the radius ratio (varying between 0.3 and 0.5), the Prandtl number (varying between 8.4 and 100) and the Taylor number. Because of external gravity the experiment can not be performed in terrestrial laboratory and is planned on the International Space Station.

A lot of authors deal with the problem of the natural convection in the spherical shells. S. Carmi and D.D. Joseph [2] calculated the critical Rayleigh numbers via linear and energy stability analysis in non-rotating case for different central force examples but not $\mathrm{r}^{-5}$. A very small gap between critical Rayleifgh number was considered.

P. H. Roberts [5] has calculated the flow and the critical Rayleigh numbers for very rapid rotation ($Ta \rightarrow \infty$) and has found the relation between critical Rayleigh number and Taylor number: $\mathrm{Ra} \sim \mathrm{Ta}^{2/3}$. This relation satisfy very good, as we will show in this paper.

F. Busse [1] predicted "columnar cells " for this limit case due to asymptotic theory. J. Hart, G. Glatzmaier and J. Toomre [3] performed the investigation of the thermal convection in a rotating hemispherical shell with radial gravity field. But they neglected the centrifugal force and therefore the results are valid for small Taylor numbers only. We investigate the same effect as in [3] but with influence of the centrifugal force, because even for small Taylor number it is important. Moreover, we perform our research in the full spherical system

In this talk we will present the results of numerical simulations, providing the design of this experiment, for example in choosing the optimal radius ratios and the angular velocity of the spherical system. The influence of the centrifugal force will investigate detailed. As work fluid the silicon oils - M1, M3, M5, M10 are using. Because of electrodynamical and dielectrical properties of these fluids they are very usefull.

This research is a numerical basis for the experiment on the ISS (International Space Station), that is preparing in the Department of Aerodynamics and Fluid Dynamics of the Brandenburg University of Technology (Germany).

## Equations and parameters

We investigate the natural convection under the radial symmetrical electrical field in the rotating spherical gap, which is filled with dielectric fluid. The two concentric spherical shells are maintained at constant, different temperatures. The innere sphere is warmer and maintained at a constant temperature $\mathrm{T}_1$ while the outer sphere is at a constant temperature $\mathrm{T}_2$. In nondimensionalized form the equations are:

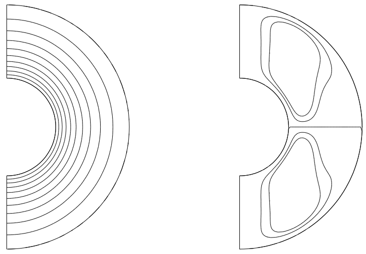


Figure 1: Basic flow: left - Temperaturfunction; right - streamfunction

$$\mathrm{Pr}^{-1}\left[\frac{\partial \mathbf{U}}{\partial t}+(\mathbf{U}\cdot\nabla)\mathbf{U}\right] = - \nabla P+\Delta\mathbf{U}+\frac{\mathrm{Ra}}{\beta^2}\frac{T}{r^5}\mathbf{e_r} - \sqrt{\mathrm{Ta}}\mathbf{e_z}\times\mathbf{U}-\mathrm{Ra}^{'}\mathrm{T}\mathbf{s} \quad (1)$$

$$\frac{\partial T}{\partial t}+\mathbf{U}\cdot\nabla\mathrm{T}=\Delta\mathrm{T} \quad (2)$$

$$\nabla\cdot\mathbf{U}=0, \quad (3)$$

with boundary conditions:

$$\mathbf{U}(\eta)=\mathbf{U}(1)=0, \qquad T(\eta)=1, \qquad T(1)=0 \quad (4)$$

The third term in equation (1) is due to dependence of the dielectrical permittivity on the temperature distribution [3].

The parameters of the problems: Prandtl number - $\mathrm{Pr}=\frac{\nu}{\kappa}$, Rayleigh number - $\mathrm{Ra}=\frac{2\epsilon_0\gamma}{\rho_0\nu k}V_{rms}^2\Delta\mathrm{T}$, Radius ratio - $\beta=\frac{\mathrm{R_2-R_1}}{\mathrm{R_1}}$, Taylor number - $\mathrm{Ta}=\left(\frac{2\Omega R_2^2}{\nu}\right)^2$, and $\mathrm{Ra}^{'}=\frac{\alpha\Delta\mathrm{T}}{4}\mathrm{TaPr}$.

In the case of non-rotating ($\mathrm{Ta}=0$) convection we have no basic flow, but radially symmettrical temperature distribution. In this case, how in the classical Rayleigh.Bénard convection, the stability investigation can be performed immediately.

The convection problem in the rotating case is much more complicated because of the basic flow, that occurs due to centrifugal force. This basic flow can be evaluated numerically only. The example of this basic flow can be seen on the Figure 1.

**Linear instability analysis and numerical method**

To write the equation for pertubation flow in frames of the linear instability theory the following functions for the velocity, temperature and pressure are introduced:

$$\mathbf{U}^{'}=\mathbf{U}+\mathbf{u}, \qquad \mathrm{T}^{'}=\mathrm{T}+\Theta, \qquad \mathrm{P}^{'}=\mathrm{P}+\mathrm{p}, \quad (5)$$

where $\mathbf{u}$, $\Theta$ and $\mathrm{p}$ are pertubation functions.

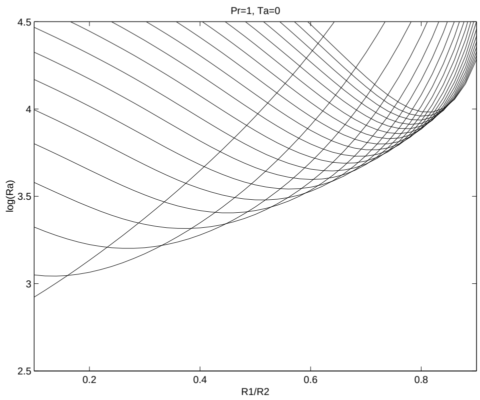


Figure 2: Stability curve: critical Rayleigh number as function of $\eta$.

After substitution of (5)in (1) - (3) the equations for the pertubation function can be written and the eigenvalue problem formulated. To integrate the equationsystem numerically a full spectral methods is used [4]. In radial direction we expand the function due to Chebyshev polynomials (truncation order $\mathrm{Nc}$) and in latitudinal and longitudinal directions due to associated Legendre functions (truncation order $\mathrm{Np}$). Because of solenoidal nature of velocity field, it can be decomposed as:

$$\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2, \tag{6}$$

where

$$\mathbf{u}_1 = \mathrm{rotrot}(\Phi \mathbf{e_r}), \qquad \mathbf{u}_2 = \mathrm{rot}(\Psi \mathbf{e_r}), \tag{7}$$

with spectral scalar functions $\Phi$ and $\Psi$, automatically satisfying (3). They are poloidal and toroidal potentials. After that the system of the vector equation can be written in terms of these scalar functions. The time stepping will perfermed due to second order Runge - Kutta method.

**Results in non-rotating case**

The calculations have been made for different $\eta = 0.1 - 0.9$ with the step $\Delta\eta = 0.02$ and $\mathrm{Pr} = 1, 10, 100$ and the stability curves are represented in Figure 2. can be seen the temperature and stream functions foe pertubation flow. The numerical research shows:

1. All eigenvalues $\sigma$ are real and the bifurcation is always steady and the principle exchange of stabilities is valid. In other words, the critical Rayleigh number depends not on Prandtl number Pr.

2. The narrower the gap is, the more is the critical Rayleigh number and higher the critical $\ell$ - mode.

3.For $\eta > 0.6$ is difficult to see the critical mode, because corresponding Rayleigh numbers are very close. Therefore for our future experimental and numerical investigations we select $\eta = 0.1, 0.2, 0.25, 0.3, 0.4, 0.5$ only.

4. For all $\eta$ the step $\tau = 0.001$ was enough to receive convergence solution.

**Results in rotating case**

We have investigated the stability of the basic flow for fluids with Prandtl numbers $\mathrm{Pr} = 8.4$, $\mathrm{Pr} = 38.96$ and $\mathrm{Pr} = 101.38$. The example of the stability research can be seen on the Figure 3. On the Figure

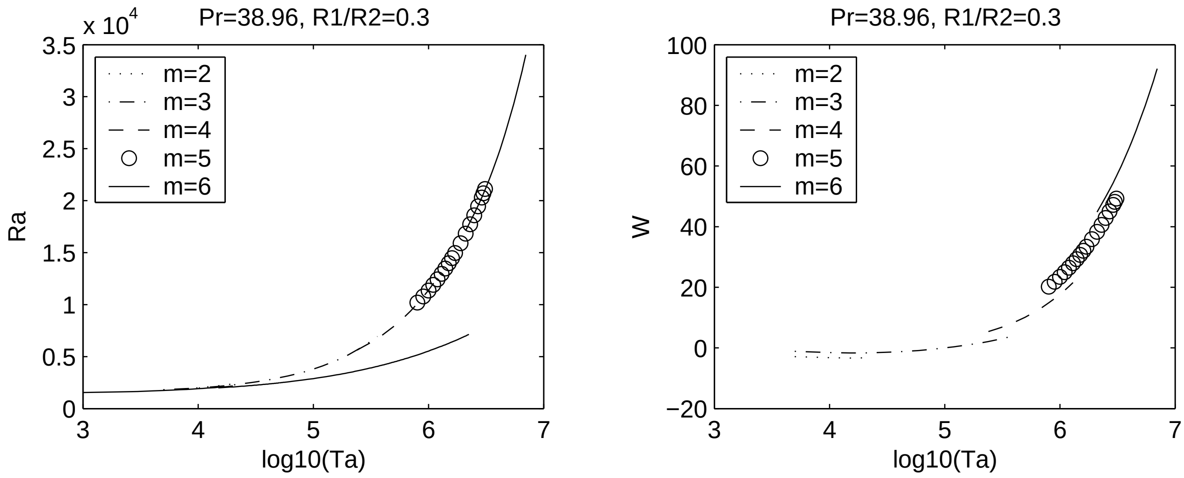


Figure 3: a - Stability curve: critical Rayleigh number as function of $\mathrm{Ta}$; b - Drift velocity as function of $\mathrm{Ta}$

3 the difference between critical Rayleigh numbers with and without influence of the centrifugal force can be considered. The curve below corresponds to the case, wenn the centrifugal force is neglected.

The stability curves (a) are represented the critical Rayleigh numbers. The curves (b) represent drift velocity of the pertubation motion.

From the stability analysis can be drawn the following conclusions:

1. By fixed Prandtl and Taylor numbers the critical Rayleigh number is bigger for narrower spherical shell.

2. The critical stability curve has similar character for all Prandtl numbers.

3. For very big Taylor numbers the results are agree very good with from Roberts forecasted law, that $\mathrm{Ra} \sim \mathrm{Ta}^{2/3}$.

4. To receive stability results with very good accuracy we need $\mathrm{Nc} = 15$ for Chebychev polynomials truncation order and $\mathrm{Np} = 30$ for Legendre polynomials truncation order; time step depends on Taylor number and to receive convergency can be varied from $10^{-4}$ to $10^{-6}$.

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