Identification of Vortex Information from Flow Field in Taylor-Couette system

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ABSTRACT

Taylor vortex flow between two-concentric accelerated cylinders with finite length is investigated numerically. The outer cylinder wall and the upper and lower end walls of the cylinders are stationary. The inner cylinder begins to rotate with constant acceleration rates. One of the characters of Taylor-Couette flow is its non-uniqueness. Even when the aspect ratio and the Reynolds number are fixed, various flow modes and transitions leading to a final flow mode appear. In this paper, the non-uniqueness of Taylor-Couette flow is investigated at wide range of the aspect ratio, Reynolds number and acceleration time of the inner cylinder. To analyze the mode transition processes, the expert system that detects the mode of the flow quantitatively is established. Mode transition process is not simple but multiple. We summarized mode transition processes at moderate aspect ratios.

INTRODUCTION

After Benjamin[1] showed an interest in Taylor-Couette flow between two concentric rotating cylinders with finite lengths, the flow has been studied as one of paradigms about a pattern transition problem. One of the major characteristics of Taylor-Couette flow is its non-uniqueness. Even though the aspect ratio, Reynolds number and radius ratio are kept constant, various modes may appear. The experimental result of Benjamin and Mullin [2] showed that multiple modes including anomalous modes appear at a given aspect ratio and Reynolds number. Some of numerical studies [3, 4, 5] on the development of Taylor-Couette flows predicted the non-uniqueness of the flow modes.

In the present study, Taylor-Couette flow is taken as an representative of the flow that shows various transition processes. We develop a heuristic and quantitative method that efficiently detects the mode of the flow, and clarify the mode transition processes when the aspect ratio, Reynolds number and the acceleration rates of the inner cylinder are varied.

NUMERICAL METHOD

The radius of the inner cylinder is r_i, the radius of the outer cylinder is r_o and the gap width between two cylinders is $D = r_o - r_i$. D is 10cm in dimensional form. The length of the cylinder is L, and the aspect ratio Γ is the fraction of the cylinder length L to the gap width D. The reference length is D, and the reference velocity is $r_i\omega_0$ where ω_0 is the maximum angular velocity attained during each calculation run. Each physical quantity is represented in the non-dimensional form by the reference values D and $r_i\omega_0$. The kinematic viscosity ν is 6×10^{-6} m^2/s. The governing equations are the axisymmetric Navier-Stokes equations and the equation of continuity in the cylindrical coordinate system (r, θ, z).

The basic solution method is the MAC method, and the convection terms are formulated by the QUICK method, and other spatial differences are given by the second-order central difference method. The time integration is the Euler explicit method.

The boundary conditions of velocity components are non-slip condition, and the pressure boundary conditions are the Neumann conditions that are estimated from the pressure terms in the momentum equations. In the initial conditions, velocity components are zero in the entire

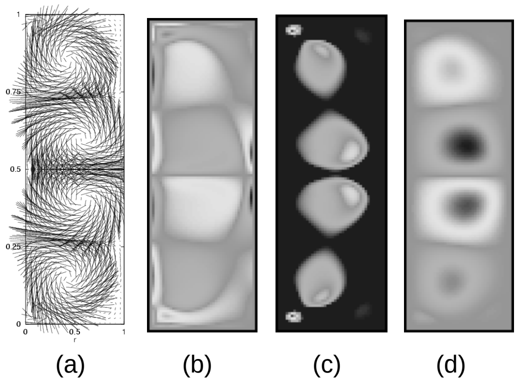
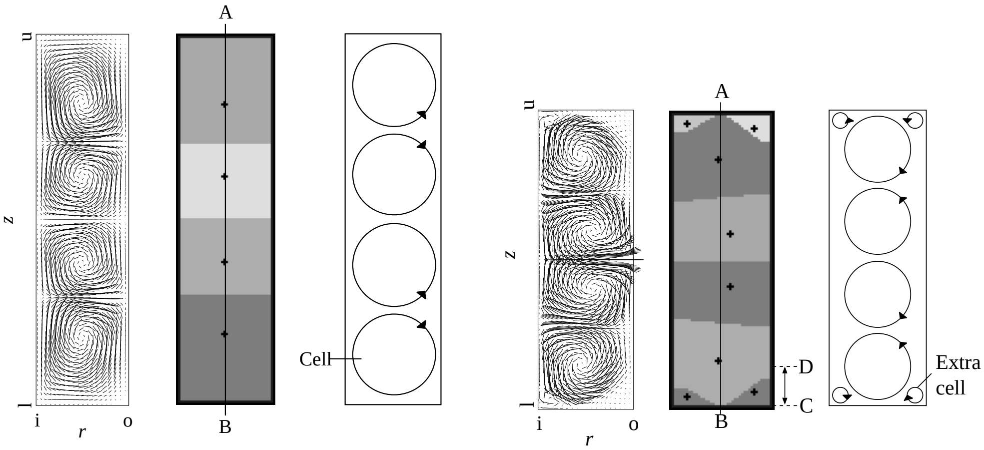


Figure 1: Various method to visualize vortices in flow field of anomalous four-cell mode. (a) Velocity vector. (b) Contour of vorticity. (c) Contour of Q invariant. (d) Contour of stream function.



(a) Velocity vector (b) Same sign areas of ψ (c) Schematic flow pattern

(i) Normal four-cell mode (Γ = 4.0, *Re* = 100)

(d) Velocity vector (e) Same sign areas of ψ (f) Schematic flow pattern

(ii) Anomalous four-cell mode (Γ = 3.2, *Re* = 400)

Figure 2: Flow patterns of Taylor vortex flow within finite annulus.

region of the flow filed. The radius ratio η is 0.667. The velocity of the inner cylinder is linearly accelerated from zero to the maximum Reynolds number attained at each calculation during an acceleration time T, then it is kept constant.

The aspect ratio is from 2.6 to 4.6 with the interval of 0.2, the Reynolds number is from 100 to 1000 with the interval of 100, T in dimensional form is from 0.0 sec to 2.0 sec with the interval of 0.2 sec, and the 1210 combinations in total are investigated.

MODE DETECTION

Benjamin[1] suggested that the mode transition of the flow that appears in the system with the inner cylinder starting from rest is very complex and it is difficult to observe in experiment. We use a domain-specific knowledge about Taylor-Couette flows and construct a quantitative detection system that automatically classifies modes of Taylor-Couette flows. In flow modes found in the present calculations, main vortices aligning in the axial direction extend across the mid-plane of the radial direction. The anomalous modes have two or four small vortices at

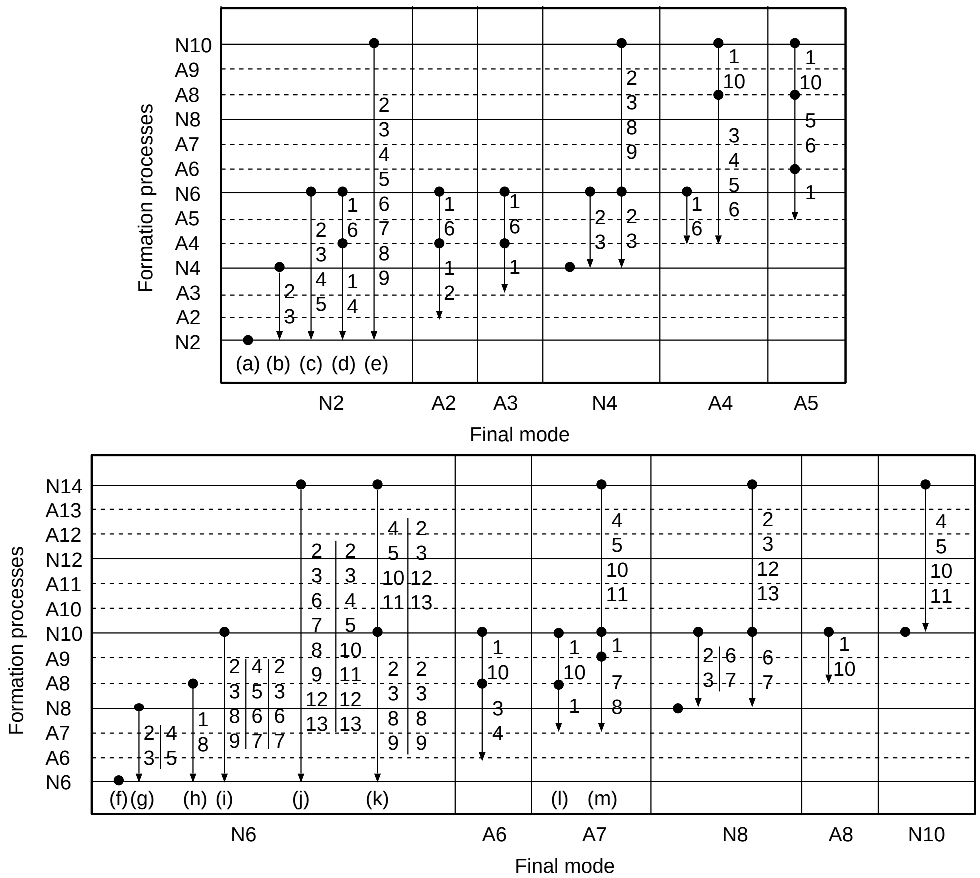


Figure 3: Mode transition processes of Taylor vortex flow.

the corners of the meridional section. These small vortices reach end walls and, as far as the anomalous modes we have observed in the present study are concerned, their heights from end walls do not exceed the half of main vortices' heights. Following these evidences, an empirical method of mode detection is introduced.

Some of the major methods for the identification of vortices use velocity vectors, vorticities, stream functions and the second invariant of velocity gradient (Q invariant)[6]. Figure 1 shows the comparison with these methods in the flow field of the anomalous four-cell mode. The abscissas is the radial coordinate (r) and the ordinate is the axial coordinate (z). The left-hand side denoted by i is the inner cylinder side and the right-hand side with o is the outer cylinder side, and u and l at the top and bottom correspond to the upper and lower end walls. The center of vortices determined from velocity vectors and those estimated by each detection method are compared, and it can be shown that the method with stream functions give the most preferable result. Thus, in the present paper, the method with stream functions is selected for the detection of vortices.

Figure 2 is used to explain the vortex detection method based on stream functions. In the Figs. 2 (a)-(f), profiles in the meridional section are shown. Figures 2 (a) and (d) shows profiles of velocity vectors of the normal four-cell mode and the anomalous four-cell mode, respectively. Figure 2 (b) and (e) show connected parts in which the signs of the stream functions are constant. In these figures, the positions where the maximal or minimal values of the stream function appear in the connected parts are plotted by cross symbols. The vortex regions and the positions of vortex center evaluated by the velocity profiles in Figs.2 (a) and (d) and the regions with constant signs of the stream function and positions of maximal values and minimal values in Figs.2 (b) and (e) show good agreement. The global vortex regions are well identified by the signs of stream function. Particularly, vortices at the four corners of the flow field, which are difficult to be recognized in the figures of velocity vectors, can be faithfully distinguished. The connected parts of constant signs of the stream function are utilized to determine the flow modes.

The important factors for the identification of flow modes are cells and extra cells. In the present paper, they are defined as follows. The lines A-B shown in Figs.2 (b) and (e) are the centerlines in the radial direction. A cell is defined as a connected part with a constant sign, which lies over the centerline. By this definition, the flows in both Figs.2 (b) and (e) have four cells. The region extending from the end wall of cylinders by a half of the average height of cells is called an end-wall region. The end-wall region in Fig.2 (e) is denoted by the region C-D. An extra cell is defined as a connected part appearing within the end-wall region. In Fig.2 (e), four connected parts at the corners between cylinder walls and end walls are determined to be extra cells. The mode of flows without extra cells is the normal mode and the mode of flows with extra cells is the anomalous mode. By using this mode detection method, flow modes of the Taylor-Couette flow are investigated.

CLASSIFICATION OF MODE TRANSITION PROCESS

Fully developed flows of the normal two, four, six, eight and ten-cell modes and the anomalous two, three, four, five, six, seven and eight-cell modes are confirmed in the present study. The transition processes of these final flow modes are classified and summarized in Fig.3. The abscissa is the final mode and the ordinate is transient mode found during the flow development from the initial quiescent state to the final mode. Here, Nn denotes the normal n-cell mode and An is the anomalous n-cell mode. Arrows in the figure show the directions of mode transition, and black circles express intermediate modes appearing during mode transitions. Numbers ns on the right side of each arrow mean that the nth vortex from the end wall disappears.

Some transition processes of the normal six-cell mode and the normal eight-cell mode have more than one column of vortices' numbers. This means that the positions of vanishing vortices are different from each other even though the transition process are identical.

CONCLUSION

Unsteady Taylor-Couette flows developing in cylinders with finite length are studied numerically. The quantitative method to determine the mode of Taylor-Couette flow is devised, and it is confirmed that this method work very well for the mode classification based on global vortices in flows. For the range of values of the aspect ratio, the Reynolds number and the acceleration time, the final modes and the mode transition processes are classified. The transition processes of each final mode are not unique, and various and multiple stages may appear during the mode transition. Disappearing vortices in a mode transition process may not be determined even though the final mode and mode transitions are fixed.

REFERENCES

[1] Benjamin, T. B. *Bifurcation phenomena in steady flows in a viscous flow - II. Experiment-*, Proc. R. Soc. Lnd., **A**, **359**, 27-43, 1979.

[2] Benjamin, T. B. and Mullin, T. *Notes on the multiplicity of flows in the Taylor experiment*, J. Fluid Mech., **121**, 219-230, 1982.

[3] Alziary de Roquefort, T. and Grillaud, G. *Computation of Taylor vortex flow by a transient implicit method*, Comput. Fluids, **6**, 259-269, 1978.

[4] Furukawa, H., et al. *Flow pattern exchange in the Taylor-Couette system with a very small aspect ratio*, Phys. Rev. E, **65**, 036306-1-7, 2002.

[5] Abshagen, J., et al. *Transient dynamics at the onset of Taylor vortices*, J. Fluid Mech., **476**, 335-343, 2003.

[6] Jeong, J., Hussain, F. *On the identification of a vortex*, J. Fluid Mech., **285**, 69-94, 1995.