Magneto-rotational instability and dynamo in hydro-magnetic Couette flow

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ABSTRACT

If the working fluid is electrically conducting, the classical Taylor-Couette problem has aspects which are relevant to astrophysics.

Here we consider two examples. In the first we study hydro magnetic Couette flow in the presence of an axially imposed magnetic field (Ruediger and Zhang 2001). We find that the applied magnetic field greatly destabilises the azimuthal Couette profile, and that the instability extends in the hydrodynamically Rayleigh-stable region. The result is important in the context of accretion discs (Balbus and Hawley 1991).

In the second example we remove the imposed magnetic field and find that the helical motion of the fluid in the Taylor vortex flow regime can excite a self-sustaining magnetic field by dynamo action. This result is relevant to the current experimental attempts to recreate in the laboratory a fundamental process of astrophysics such as a dynamo (Gailitis et al 2001; Stieglitz and Mueller 2001).

Magneto-rotational instability

We consider two concentric cylinders of inner radius R_1, outer radius R_2 and infinite length which rotate at constant angular velocities Ω_1 and Ω_2. The governing equations of motion for the velocity field $\mathbf{v}(r, \phi, z, t)$, the pressure $p(r, \phi, z, t)$ and the magnetic field $\mathbf{b}(r, \phi, z, t)$ are

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla p + \nabla^2 \mathbf{v} + \frac{Q}{P_m}(\nabla \times \mathbf{b}) \times \mathbf{b}, \tag{1}$$

$$\frac{\partial \mathbf{b}}{\partial t} = \frac{1}{P_m}\nabla^2 \mathbf{b} + \nabla \times (\mathbf{v} \times \mathbf{b}), \tag{2}$$

$$\nabla \cdot \mathbf{v} = \nabla \cdot \mathbf{b} = 0, \tag{3}$$

The dimensionless parameters of the problem are the Reynolds numbers

$$Re_1 = \frac{R_1 \Omega_1 \delta}{\nu}, \tag{4}$$

$$Re_2 = \frac{R_2 \Omega_2 \delta}{\nu}, \tag{5}$$

the radius ratio

$$\eta = \frac{R_1}{R_2}, \tag{6}$$

the angular velocity ratio

$$\mu = \frac{\Omega_2}{\Omega_1}, \tag{7}$$

the Chandrasekhar number

$$Q = \frac{B_0^2 \delta^2 \sigma}{\rho \nu}, \tag{8}$$

and the magnetic Prandtl number

$$P_m = \frac{\nu}{\lambda}, \tag{9}$$

In writing these equations we used $\delta = R_2 - R_1$ as unit of length, δ^2/ν as unit of time, and B_0 (the strength of the applied magnetic field) as the unit of magnetic field. The fluid contained between the cylinders is characterised by constant density ρ, kinematic viscosity ν, magnetic diffusivity $\lambda = 1/(\sigma\mu_0)$, electrical conductivity σ and permeability μ_0. The equations are solved using a spectral method (Willis and Barenghi 2002a) assuming no-slip boundary conditions for $\mathbf{v}$ and electrically insulating boundary conditions for $\mathbf{b}$.

It is well know that in the absence of magnetic field inviscid Couette flow is linearly stable if the Rayleigh criterion $\mu > \eta^2$ is satisfied. Figure 1 shows the critical Reynolds number Re_{1c} of the inner cylinder versus Re_2 at different values Q of the applied magnetic field. It is apparent that if Q is large enough a flow which would be Rayleigh stable becomes unstable. The symmetry $e^{im\phi}$ of the most unstable mode (to which the figure refers to) is $m = 0$.

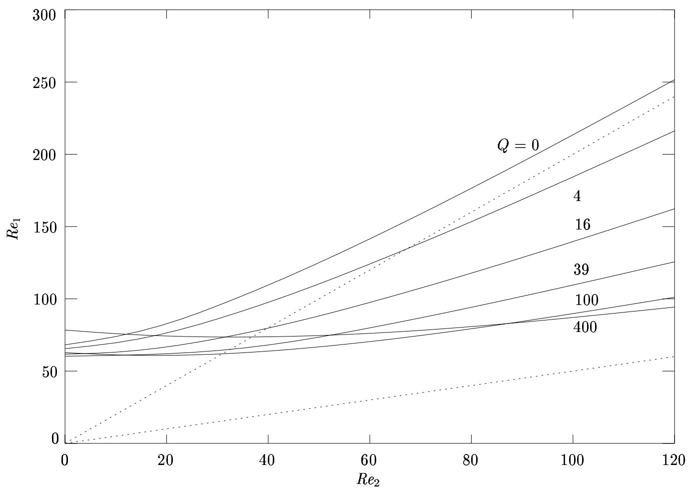

Figure 1: Re_{1c} vs Re_2 at $\eta = 0.5$ and $P_m = 1$ at different values of Q. The upper dotted line is Rayleigh's criterion, the lower dotted line is solid body rotation

The destabilising effect of the magnetic field is larger the larger P_m is (Willis and Barenghi, 2002b), as shown in Figure 2. The significance of this result is due to the possibility that large values of P_m exist in central regions of galaxies (Kulsrud and Anderson 1992).

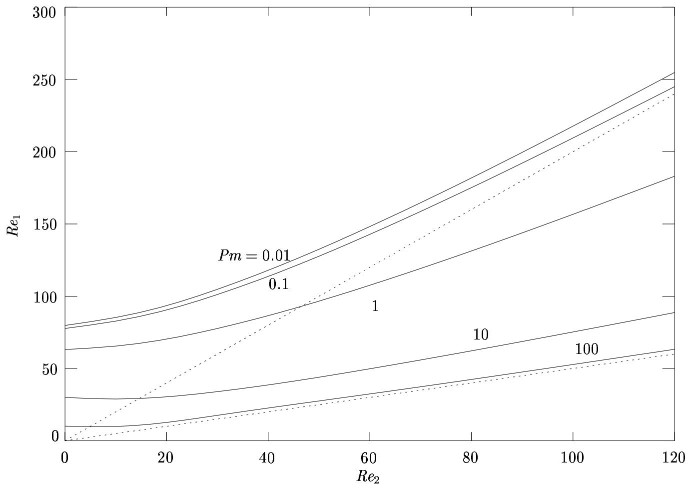

Figure 2: Re_{1c} vs Re_2 at $\eta = 0.5$ and $Q = 10$ at different values of P_m. The upper dotted line is Rayleigh's criterion, the lower dotted line is solid body rotation

Dynamo action

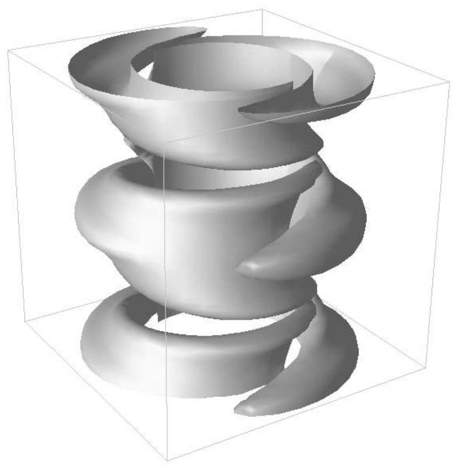

Figure 3: Isosurfaces of $|\mathbf{b}|$ for $\eta = 0.5$, $\mu = 0$ and $P_m = 2$.

We repeat the calculation without the imposed magnetic field. The same dimensionless parameters are used as before, but now the unit of magnetic field is $(\mu_0 \rho)^{1/2} \nu / \delta$. First we solve Eq. 1 and create a steady axisymmetric flow at given values of Re_2 and $Re_1 > Re_{1c}$ where Re_{1c} is the critical velocity for the onset of Taylor vortex flow. Then we integrate Eq. 2 and study if the magnetic field, driven by the imposed Taylor vortex flow, grows or decays. We find that the most favourite mode for the growth of magnetic field is $m = 1$ and that the largest growth rate is obtained with some co-rotation. Finally we integrate Eq. 1 and 2 together, and show that it is possible to achieve a saturated self-consistent dynamo (Willis and Barenghi 2002c). Figure 3 shows the isosurface of $|\mathbf{b}|$. Most of the magnetic energy is in the azimuthal $m = 1$ mode, with less energy at higher (odd) values of m. Similarly, most of the kinetic energy is in the $m = 0$ mode, with less energy at higher (even) values of m. Figure 4 shows that the magnetic energy (in units of the energy E_{CCF} of the circular Couette flow) initially grows exponentially with time and then achieves saturation. The curves (a) and (b) refer to rotation of the inner cylinder only and co-rotation respectively.

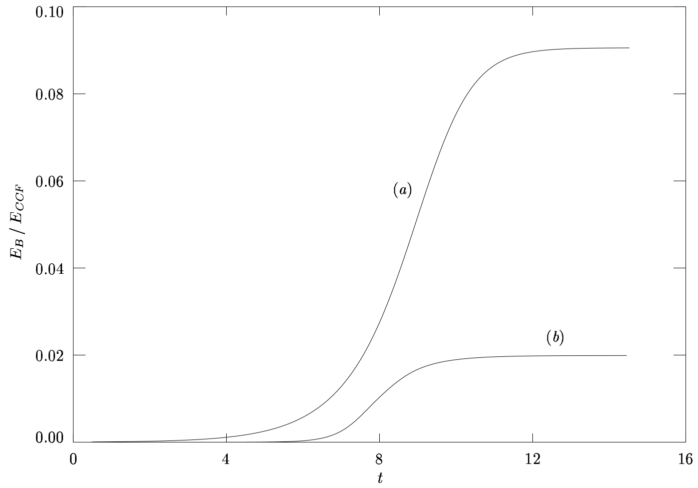

Figure 4: Magnetic energy versus time in units of E_{CCF} for $\eta = 0.5$ and $P_m = 2$. (a): $\mu = 0$, $Re_1 = 1.5Re_{1c}$ and $E_{CCF} = 4.15 \times 10^4$; (b): $\mu = 3\eta^2/4$, $Re_1 = 2Re_{1c}$ and $E_{CCF} = 3.63 \times 10^5$.

Conclusion

Hopefully these results will stimulate more work on the nonlinear saturation of the magneto-rotational instability and on dynamos at the very small values of P_m relative to experiments done in the laboratory.

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