An asymmetrical periodic vortical structures and appearance of the self-induced pressure gradient in the modified Taylor flow.

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ABSTRACT

An incompressible liquid flow in the gap between two coaxial cylinders, such that the inner rotating (wavy) cylinder has a periodically varying radius along the axial direction while the outer stationary cylinder has a constant radius, is studied experimentally and theoretically. The basic attention is focused on the symmetry-breaking phenomenon of the vortex flow arising from the rotation of the inner wavy cylinder. It is found that the symmetry-breaking phenomenon of the vortical flow structures in this geometry is accompanied by occurrence of a self-induced axial pressure gradient. A generalized formulation of the problem of periodic vortical flow prevailing in the gap between two coaxial wavy cylinders having large axial length is presented. The comparison between the computed and the experimental results is presented and the underlying phenomena are discussed.

INTRODUCTION

An incompressible liquid flow between two coaxial cylinders, arising from the rotation of the inner cylinder, is well known in hydrodynamics as Taylor-Couette flow (see [1]). The modified Taylor flow is realized in geometry, where one or both rigid surfaces have axisymmetric wavy shape due to periodically varying radius along the axis of rotation. The existence of such flows, caused by the wavy surface of the rotor was investigated in the works [2] (1999) and [3] (2001). The modified Taylor flow, with both inner and outer cylinders having a wavy shape, has been analyzed numerically in [4] (2002).

In the present work the experimental and numerical investigations of the case, where the fixed external cylinder has a constant radius, while the radius of the inner rotating cylinder varies along the axial direction following a cosine law, are presented. The attention is focused on the symmetry-breaking phenomenon of the vortical flow, which is accompanied by occurrence of a self-induced axial pressure gradient. The symmetry-breaking phenomenon of the flow in the region having geometrical symmetry has been scrutinized in a number of studies. The existence of periodic flow in the non-periodic pressure field with the given gradient is well known too. But the possibility of occurrence of a self-induced pressure gradient in the periodic flow field of an incompressible liquid in a closed region having large axial lengths has never been studied. In the present work this phenomenon is investigated theoretically and a generalized formulation of the problem of periodic vortical flow calculation in the region with the large axial lengths is presented. The computed results are compared with experimental observations and measurements.

1. Experimental apparatus, the methods and means of experimental investigation.

We use the same experimental apparatus as in the work [2]. It consists of the transparent outer cylinder with the radius R=64.11 mm and the length H=263 mm. Inside the outer cylinder there is a coaxial rotating cylinder representing one of 6 bodies of rotation. The geometrical parameters of the variety of inner cylinders (rotor) used in this study are given in Table-1. Experiments are performed using one cylinder having a constant radius (CR1) and five cylinders (WR2,…,WR6) with periodically varying radius along the axial direction, as described by the formula:

$$R_1(z)=a_0+a_1\cos(2\pi z/\lambda) \qquad (1)$$

Table-1

rotor	(Made dimensionless by R)		
	a_0	a_1	λ
CR1	0.8002	0	-
WR2	0.8002	0.1221	1.1192
WR3	0.8002	0.1221	0.5597
WR4	0.8002	0.0610	1.1192
WR5	0.8002	0.0610	0.5597
WR6	0.8002	0.0306	1.1192

Hereinafter the internal radius (R) of the cylindrical container is used as characteristic dimension.

The methods and means employed for experimental investigations consist of: (i) Measurement of rotor rotation speed in the range 30-1000 rpm with an error less than ±3 rpm, (ii) measurement of temperature of the liquid in the range 15-40 ^{0}C; (iii) visualizations and video recording of the flow pattern at the outer cylindrical wall by the kaliroscope particles; (iv) visualizations and video recording of the flow pattern in (r,z)-plane using a laser sheet with the kaliroscope particles; (v) measurements of velocity distribution (r,z)-plane using Particle Image Velocimetry (PIV) and (vi) measurement of a stationary pressure difference between two points located on the outer cylinder (error < ±0.1 mm of water). As a main parameter determining the flow regime the Taylor number is used:

$$T = \mathrm{Re}\sqrt{a_0}(1-a_0)^{3/2} \quad ; \quad \mathrm{Re} = \frac{\rho\omega R^2}{\mu}. \tag{2}$$

Where a_0 is the dimensionless average radius of the rotating cylinder, μ=[0.045-0.065] Pa/s is the viscosity and ρ=1.027g/cm^3 is the density of the liquid in the gap and ω is the angular velocity of the rotor.

2. Mathematical model and numerical methods.

Consider the region confined between the inner rotating and outer stationary cylinders. Let the axial length H of the region along the z-axis be much greater than the radius R of the external cylinder and the wavelength λ of the rotor surface (H >> R; H >> λ). Let the gap between the cylinders be filled with an incompressible viscous liquid having constant viscosity. A cylindrical coordinates system centered at the axis of rotation of the inner cylinder, such that the radial axis *r* passes through one of the maxima of the wavy surface, and is approximately equidistant from the left end *A* and right end *B* of the fluid column, is introduced.

The mathematical model considers time-independent axisymmetric Navier-Stokes equations. Velocity, and pressure scales are defined as: $V_s = \omega R$; $P_s = \rho(\omega R)^2$. The geometrical parameters are made dimensionless by R. Although, the mathematical model considers a general case of the modified Taylor geometry with both wavy surfaces, the shapes of which can be described by any continuous functions *$R_1(z)$, $R_2(z)$* with the identical period $\lambda=2\pi/s$, in the present work a special case where the outer cylinder having a constant radius (R_2=1) is considered. Without loss of generality, the pressure can be represented by:

$$\hat{P} = P_0 + \rho(\omega R)^2[P(r,z) - G\cdot z] \tag{3}$$

Where P(r,z,t) remains a bounded function for an infinite increase of region length H.

The no-slip boundary conditions are applied at the rotor and the fixed cylinders,

$$r=R_1(z):\ u=v=0,\ w=R_1(z)\ ;\ r=1:\ u=v=w=0 \tag{4}$$

and the same conditions are applied on the axial (left *A* and right *B*) ends of the region,

$$z=Z_A,\ z=Z_B:\ u=v=w=0 \tag{5}$$

where v, w, u are the radial, azimuthal and axial velocity components, respectively. Besides it is necessary to set a value of pressure in the one arbitrary point in the region.

$$\hat{P}(1,0) = P_0 \quad \text{or} \quad P(1,0) = 0 \tag{6}$$

The classical statement of a problem for the movement of an incompressible liquid in the closed region described above does not depend on the region size H and provides the locally unique stationary solutions, for at least small Taylor numbers T. The local uniqueness of solutions can be broken only at bifurcation points.

We assume periodicity of all variables along the z-axis with wavelength λ (wave number s) and represent the solution in a form of a Fourier series. The Fourier coefficients of non-linear terms from Navier-Stokes equations are obtained in an analytical form. The system of differential equations containing Fourier coefficients is solved by the finite differences method. The calculation domain is discretized using 12-25 harmonics per wavelength and 41-81 points on the radial coordinate.

3. The analysis of investigation results

Symmetric stationary vortical flow: Experiments performed using 6 different rotating inner cylinders as listed in Table-1 and the flow regimes in the range from T=0 to T= 600 are explored. The numerical analyses are performed for all variants of the rotor geometry and majority of stationary flow regimes existing in the range of T=0 to T=500 are explored. In addition, at T=84 and T=325 the stationary flow is investigated with a continuous variation of the parameters $0.03 < a_1 < 0.15$, $0.34 < \lambda < 0.97$.

The experimentally observed and numerically computed flow patterns and velocity field shows a good qualitative and quantitative agreement for stationary regimes both. In Fig.1 the experimental flow pattern in the (r,z)-plane using the rotor WR5 at T=210 is shown. From Fig.1, one deduces that unlike the classical Taylor flow, the modified Taylor flow is two-dimensional and axisymmetric even at small T<<1. It has a fixed wavelength, which is equal (or a multiple) to the wavelength λ of the rotor surface. The investigations have shown that, for small and moderate values of T, the stationary vortical structures follow the symmetry of flow region, if such symmetry is present. Here one wavelength consists of two symmetric vortices with opposite circulation. The presence of two vortices in one wavelength is not a strict property. For T>55, using rotors WR4, WR6 the symmetric flow with 4 vortices per wavelength are observed, and the branch of the solutions with 6 vortices per wavelength is found in calculations. In the tests with rotors WR3, WR5, the stationary periodic flows are obtained up to T=400, which indicates the large stability of such flows. A similar result has been found in the work [3]. Even after intense pulsations appear at large T, the modified Taylor flow continues to keep the large-scale vortical structure pertaining to the geometry of the region.

Asymmetric stationary vortical flow: One of the most interesting and unexpected results of experimental studies is the detection of stationary, asymmetric vortical structures in (r,z)-plane using the rotor WR5. These structures arose from symmetric vortices at Taylor number greater than a critical value T*≅250. In Fig.2 the flow pattern observed with rotor WR5 at T=325.3 is shown. In comparison with the Fig.1, here one vortex became 1.5 times greater than the other.

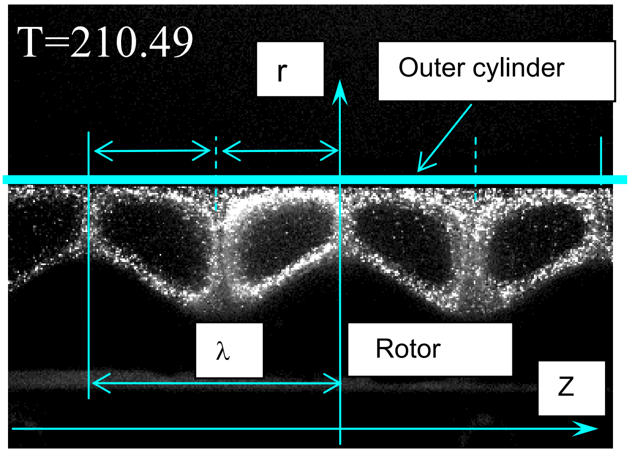

Fig.1 The symmetric flow in the (r,z) plane for the rotor WR5 at T= 210.5 (experiment).

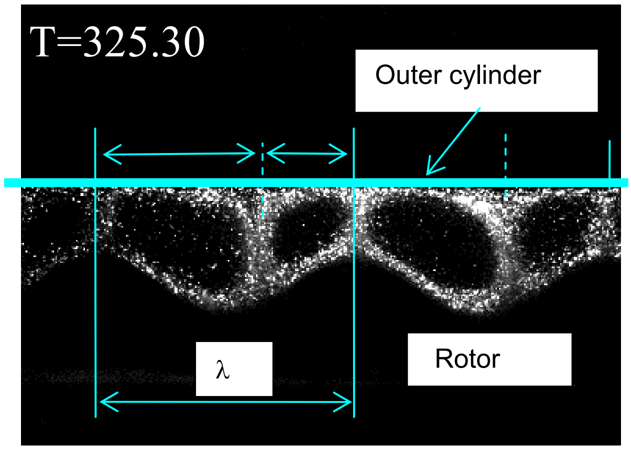

Fig.2 The asymmetric flow in the (r,z) plane for the rotor WR5 at T= 325.3 (experiment).

After reconsideration of the given problem, we made the assumption that the asymmetric vortical flow arises from the symmetrical one as a result of symmetry breaking bifurcation at the certain Taylor number T*. The Navier-Stokes equations have such asymmetric solutions, which can be found numerically.

Assuming that for some range of Taylor numbers the established flow in the region consists of the main part having the length L>>1 and buffer zones attached to ends *A, B*, having limited size a~b ~1. In the main part of region the flow is periodic with the period equal to λ or its integer number Λ=mλ. In the buffer zones the deviations from the flow periodicity and geometric symmetry are possible.

Proceeding from the classical statement of the problem of incompressible fluid flow, we formulate the problem of seeking the periodic solutions in the main part of the region by assuming that the liquid motion is governed by the Navier-Stokes equations with the no-slip boundary conditions (4) on the inner and outer surfaces, however, instead of conditions on the ends *A,* and *B* the condition of periodicity of the velocity vector is used, which as a consequence of searching the periodic solutions is generalized up to the requirement of periodicity of all variables in the Navier-Stokes equations, along with equation (6) to set pressure at an arbitrary point. It is to be noted that the Navier-Stokes system contains a parameter G due to pressure representation in the form (3). In the case of the closed borders *A, B* (5) the external pumping is absent and the axial pressure gradient is G=0. But, conceptually by decomposing the solution domain into the main part and the buffer zones, parameter G becomes uncertain and the problem becomes ill posed. As the Navier-Stokes equation for whole region is completely determined, it should contain a closing condition, concerning the periodic flow in the main part. Such a condition, comes from the ends *A* and *B* of region, and is the condition of zero net axial flux through any cross section of the region.

$$Qz = 2\pi \int_{R_1}^{1} u(r,z) \cdot r \cdot dr = 0 \qquad (7)$$

We emphasize that for correct formulation of the problem, the initially given condition is the zero net axial flux (7), but the pressure gradient G is dependent parameter to be searched for along with the rest of dependent variables. With this formulation the asymmetric stationary periodic solution is obtained (T>T*) in the first attempt of calculation. Agreement of the calculated flow lines and experimental trajectories of particles is very good.

The most important property of asymmetric vortex flows is the self-induced pressure gradient, the dimensionless value of which under conditions of Fig.2 is G=0.0089. The presence of the pressure gradient means that in such flows the periodic field of a velocity vector coexists with the non-periodic pressure field (3). Which implies that in the experiment there should be a pressure difference between the appropriate points separated from each other by an integer number wavelength λ. For verification, the pressure is measured at two points separated from each other by 4λ.

The tests have shown that approximately at T=230 the pressure difference begins to grow quickly and T=335 it reaches a value of 20.1 mm.

In Fig.3 the results of these tests are given in the dimensionless form, together with a calculated bifurcation curve of the self-induced pressure gradient. The agreement of these results is good.

These analyses show that the asymmetric solutions have the property of continuous dependence on the parameters of the problem. The calculations also show that the asymmetric branch of the solutions exists at least up to T=550.

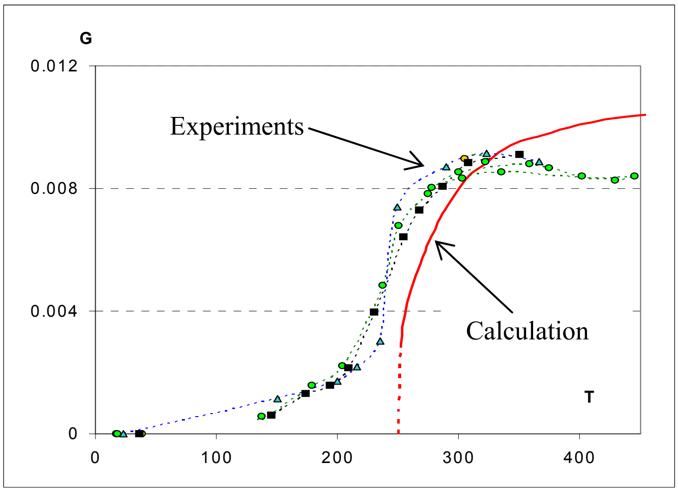

Fig.3 Self induced pressure gradient G for the rotor WR5.

Conclusions

The experimental and numerical investigation presented here allows making the following conclusions: Unlike the classical Taylor flow, the modified Taylor flow is two-dimensional and axisymmetric even at small Taylor number T<<1. It has a fixed period, which is equal (or multiple) to the wavelength λ of the rotor surface. In the modified Taylor flow at small and moderate values T, the stationary vortical structures follow the symmetry of the flow region, if such symmetry is present. The stationary regimes of the modified Taylor flow, as a rule, keep the stability in wider range of Taylor numbers than classical Taylor flow. In the modified Taylor flow with the symmetric shape of internal region geometry, the asymmetric periodical stationary vortical structures can exists, which arises from the symmetric solution, as a result of symmetry breaking bifurcation at certain critical Taylor number T*. This bifurcation is accompanied by occurrence of a self-induced axial pressure gradient, but the net axial flux remains zero. The Navier-Stokes equations have asymmetric stationary periodic solutions in symmetric region with the arbitrary length H>>1. Such solutions exist, these are stable and they continuously depend on problem parameters in the wide range of their variation. The generalization of the problem for the periodic vortical flow calculation in the region with the large axial length includes the pressure representation as the sum of function periodic along z-axis and gradient member $G(t)\cdot z$, which is unknown and should be calculated together with other dependent variables.

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