Influence of aspect ratio in thermal convection in a cylindrical annulus

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ABSTRACT

Instabilities and pattern formation in buoyant-thermocapillary flows have been extensively studied in the last years. Classically heat is applied uniformly from below [1] where the conductive solution becomes unstable for temperature gradients beyond a certain threshold. A more general set-up considers thermoconvective instabilities when a basic dynamic flow is imposed through non-zero horizontal temperature gradients [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. In [10, 13, 14] results on this problem are obtained where the importance of heat related parameters to develop the instabilities is addressed. In this paper we study the influence of aspect ratio, i.e., the relation between the radii of the box and the depth of the fluid, on the bifurcation transitions. As in [10, 13, 14] we use the primitive variables formulation, solving these equations by expanding the fields with Chebyshev polynomials (see Ref. [15]).

The physical set-up consists of a horizontal fluid layer of depth d (z coordinate) is in a container limited by two concentric cylinders of radii a and $a + \delta$ (r coordinate). The bottom plate is rigid and a linear temperature profile is imposed where the inner part has a temperature $T_{\max}$ whereas the outer one is at $T_{\min}$. The top is open to the atmosphere whose temperature is T_0. In the equations governing the system u_r, u_ϕ and u_z are the components of the velocity field u, Θ is the temperature, p is the pressure, $\mathbf{r}$ is the radio vector and t is the time. The system evolves according to momentum and mass balance equations and to the energy conservation principle, which in dimensionless form are (see Ref. [13]),

$$\nabla \cdot u = 0, \tag{1}$$

$$\partial_t \Theta + u \cdot \nabla \Theta = \nabla^2 \Theta, \tag{2}$$

$$\partial_t u + (u \cdot \nabla)\, u = Pr \left(-\nabla p + \nabla^2 u + \frac{R\rho}{\alpha \rho_0 \Delta T} e_z \right) \tag{3}$$

where the operators and fields are expressed in cylindrical coordinates and the Oberbeck-Bousinesq approximation has been used. Here e_z is the unit vector in the z direction, ρ is the density, α is the thermal expansion coefficient and ρ_0 is the mean density. The following dimensionless numbers have been introduced: the Prandtl number $Pr = \nu/\kappa$ and the Rayleigh number $R = g\alpha \Delta T d^3/\kappa\nu$, which represents the buoyant effect. In these definitions ν is the kinematic viscosity of the liquid, κ is the thermal diffusivity, g is the gravity constant and $\Delta T = T_{\max} - T_0$.

The boundary conditions (bc) are: on z =1,

$$u_z = \frac{\partial u_r}{\partial z} + M \frac{\partial \Theta}{\partial r} = \frac{\partial u_\phi}{\partial z} + \frac{M}{r} \frac{\partial \Theta}{\partial \phi} = \frac{\partial \Theta}{\partial z} + B\Theta = 0 \tag{4}$$

on z =0, $u_r = u_\phi = u_z = 0$, $\Theta = \left(-\frac{r}{\delta^*} + \frac{a}{\delta}\right)\frac{\Delta T_h}{\Delta T} + 1$,

on $r = a^*$, $r = a^* + \delta^*$, $u_r = u_\phi = u_z = 0$, $\partial_r\Theta = 0$.

Here B is the Biot number which quantifies the heat exchange with the atmosphere, $a^* = a/d$, $\delta^* = \delta/d$, $\Delta T_h = T_{\max} - T_{\min}$ and $M = \gamma \Delta T d/(\kappa\nu\rho_0)$ is the Marangoni number which includes the surface tension coefficient γ. The only control parameter mentioned in Refs. [2, 5, 6, 7, 8, 9, 12] is ΔT_h, however as discussed in [10, 13, 14] we find a new one, ΔT, also related to temperature and we prove the significance of the Biot number. In this paper we show the influence of aspect ratio δ^* on the bifurcation transitions.

Basic state and linear stability analysis. The horizontal temperature gradient at the bottom settles in a stationary convective motion, which is computated as in Ref. [13, 14]. Increasing the control parameter ΔT, makes the basic flow unstable and different bifurcations arise. The linear stability analysis supplies information on the critical values of ΔT at which this happens and on the shape of growing instabilities. We study the stability by perturbing the basic solutions with fields depending on r, ϕ and z coordinates in a fully 3D analysis, following the numerical scheme of Refs. [13, 16]. Due to the periodical boundary conditions in the azimuthal coordinate the perturbations of any physical function X can be factorized, and along ϕ is expanded by Fourier modes, $X(r, \phi, z, t) = X(r, z)e^{im\phi+\lambda t}$, where m is the wavenumber. The eigenvalue λ characterizes the instability, when its real part is negative the basic state is stable but if it is positive the basic solution is unstable. In this case the imaginary part of λ can be zero and then the bifurcation is stationary while if it is non zero the bifurcation is oscillatory.

The bifurcations obtained when ΔT is increased can be very diverse, depending on the values of the heat related quantities ΔT_h and B. We fix Prandtl number ($Pr = \infty$). This assumption is quite standard and accurately represent any fluid with Prandtl number above 10. In Refs. [13, 14] different kinds of spatially extended and localized structures appear which are both stationary or oscillatory. Also competing solutions at codimension two bifurcation (CTB) points have been found: stationary radial rolls with different wave numbers and radial rolls with hydrothermal waves. These CTB depend on the heat exchange with the atmosphere parameter (the Biot number).

Numerical Results. The aspect ratio δ^* is varied in the interval $(2.5, 12.0)$. Three types of transitions are observed: OE: from oscillatory to stationary bifurcation through a codimension two bifurcation oscillatory-estationary; EE: between two stationary bifurcations with different wave number through a codimension two bifurcation stationary-stationary; EO: from stationary to oscillatory, not through a codimension two bifurcation. Varying δ^* we find three differents scenes of transtition between bifurcations.

Large δ^* ($\delta^* = 12$ is the maximum we can reach due to the convergence of the numerical method). This region is characterized by the presence of a transition between a stationary bifurcation and an oscillatory one by changing the Biot number and through codimension two bifurcations.

Medium $\delta^*(5-10)$. There are two types of transitions, depending on ΔT_h, one from oscillatory to stationary (OE) and another one between stationary bifurcations with different wave numbers (EE). Both transitions occur by changing the Biot number. For small ΔT_h the transition is OE and for large ΔT_h it is EE. As δ^* decreases the region of transition OE lessens till it disappears for small δ^*.

Small δ^*. In this case the transition EO is obtained varying ΔT_h, the Biot number has not any influence in the bifurcation. The transition does not accour through codimension two bifurcations.

Remarkably our results recover many features of numerous reported experiments, i.e. those in Refs. [18, 17].

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