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Nonlinear mechanics of wavy instability of steady longitudinal vortices in free mixing and wall bounded flows

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In previous Couette-Taylor Workshops Girgis & Liu (1997, 1999) discussed results for the nonlinear development of steady Taylor-Görtler vortices in the free mixing region after their release from upstream wall bounded flow. These steady longitudinal vortices in turn, are susceptable to time-dependent wavy instabilities that are sometimes referred to as secondary instabilities (e.g. Yu & Liu 1991, 1994; Girgis & Liu 2002). They are akinned to the unstable wavy vortices in a Taylor-Couette flow apparatus. At this Workshop we will discuss the nonlinear mechanics of time-dependent instabilities of steady three-dimensional longitudinal vortices from a more general point of view: the steady mean flow could be Taylor-Görtler vortices or longitudinal vortices artificially generated from periodic deformities of the trailing edge as would be found in mixing enhancement or noise supression nozzle devices.

It is much more physically illuminating to carry out the analysis in physical rather than spectral coordinates. The flow is split into a time-independent mean flow and the time-dependent 'high frequency' instabilities; these are nonlinearly coupled via the Reynolds stresses of the unsteady component of flow . The total steady flow consists of longitudinal vortices and the basic steady shear flow. The mechanics of nonlinear interactions between the steady flow components are recoverable via spanwise averaging. The steady flow component continues to be parabolic in its streamwise development in the viscous sense and devoid of curvature effects, except for the Görtler mechnism, under the assumption of thin boundary and shear layers relative to the radius of curvature. The nonlinear wavy disturbance equations in physical variables are obtained by subtracting the Reynolds-averaged steady flow equations from the total flow quantities before the Reynolds splitting. The nonlinear disturbance equations are thus similar in form as in nonlinear hydrodynamic stability studies (such as Stuart 1956). Again, for small relative boundary or shear layer thickness, curvature effects in advective and viscous diffusion effects are negligible; the Görtler centrigual mechanism do not arise in the wavy disturbance problem as was found in the linear problem (Sabry, Yu & Liu 1990, Yu & Liu 1991, Hall & Horseman 1991). Arguments extracted from experiments (e.g., Swearingen & Blackwelder 1987) show that the wavy instability velocities, unlike that of the steady flow, are very nearly isotropic; their physical wavy characteristics scale isotropically according to the local shear layer dimensions while their three-dimensional wave envelope is streamwise parabolic, consistent with the steady mean flow. A number of simplifications are thus possible, leading to simpler understanding of the partial differential equations for the disturbances in the present case. The mean flow advection of disturbance momentum is attributed only to the streamwise mean flow velocity. Therein lies the hidden nonlinearity due to mean flow modification by the nonlinear disturbances. Two dominant mechanisms for instability arises: one is the normal (or vertical) disturbance velocity advection of the three-dimensional structure of spanwise mean flow vorticity, a mechanism familiar in two-dimensional shear flow instabilities (e.g., Lin 1955); the other is the spanwise disturbance velocity advection of the three-dimensional normal or vertical mean flow vorticity. These mechanisms were well identified in earlier linear studies (Yu & Liu 1991, 1994) and they remain so with nonlinearities taken into account. The nonlinear generation of higher harmonics are due to the time-dependent excess-Reynolds stresses familar in nonlinear hydrodynamic stability studies.

The system consists of the downstream marching solution of the three-dimensional longitudinal vortex mean flow and of the coupled parobolic partial differential equations for the three-dimensional wave envelope of spectral components of the disturbances, starting from imposed upstream initial conditions. Although the details are very much different, the physical implications of previous work on nonlinear hydrodynamic stability (e.g., Stuart 1960) are inferable from the present strongly nonlinear studies, which include the modification of the mean flow, the generation of higher harmonics and the effect of these on modifications of the fundamental component. These effects are distilled from the partial differential equations in the present case, reflecting the basic mechanisms depicted by respectively the Stuart constants k_1, k_2, k_3 in the Stuart-Landau amplitude equation (Stuart 1960) which were rationally derived from the Navier-Stokes equations under the assumption of weak nonlinearity for parallel flow instabilities. Some preliminary results will be shown and are used to interpret experiments (e.g., McCormick & Bennett 1994), particularly in terms of the role of sinuous and varicose modes of the wavy instabilities and their contribution to the Reynolds stresses and energy conversion mechanisms.

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