Centrifugal Instability in the Backward-Facing Step

Jean-François Beaudoin[1-2], Olivier Cadot[1], Jean-Luc Aider[2] & José Eduardo Wesfreid[1]*

1: Laboratoire de Physique et Mécanique des Milieux Hétérogènes UMR 7636-CNRS
Ecole Supérieure de Physique et Chimie Industrielles, 10 rue Vauquelin 75231 Paris Cedex 5 - France
2: PSA Peugeot Citroën, Direction de la Recherche
Centre Technique de Vélizy, Route de Gisy 78943 Vélizy-Villacoublay Cedex - France

Three-dimensional stationary structure of the flow over a backward-facing step is observed experimentally showing a periodicity in the spanwise direction., with longitudinal vortices. A local Rayleigh discriminant and a local Görtler number are computed from numerical simulations. It is shown that the observed instability is consistent with a centrifugal instability raising just downstream the recirculation zone.

1. Introduction

We are generally interested in the origin of the three-dimensionality occurring in separated flows and this communication focuses on experimental work about stationary three-dimensional aspect of the flow over a backward-facing step.

The backward-facing step is one of the simplest geometry to study the phenomenon of separation in flows. As a major benchmark for two-dimensional numerical simulations the backward-facing step has been the subject of many experimental [1] and numerical investigations [2]

Only few studies exhibit the three-dimensional aspects of this flow, especially in the steady regime. Armaly et al. [1] and Williams & Baker [3] focused on the side-wall effects experimentally and numerically. More recently linear stability analysis of Barkley et al.[4], based on numerical simulations, shows that the stationary two-dimensional flow over a backward-facing step is not stable above a certain Reynolds number. They actually predict a stationary three-dimensional intrinsic instability.

2. Experimental set up

The flow is produced by gravity in a horizontal water tunnel. The test channel has a cross-section of 10*15 cm_ and a total length of 82 cm, which allows visualizations and measurements far downstream. The mean flow velocity can range from 0.2 to 20 cm.s^{-1}. The step geometry is shown in Figure 1; it is composed of a ramp of angle 9.5° upstream a backward-facing step of height h. The Reynolds number is based on the step height h and the maximum velocity of the step edge profile, U_0. With this definition the Reynolds number ranges from 10 to 300 in the present study.

The flow is visualized by means of Laser induced fluorescence (LIF) in different planes x = cst and z = cst. The dye injection is performed in the upstream boundary layer through 50 holes of 0.7 mm of diameter. The velocity field is measured with a PIV set-up in different planes z = cst.

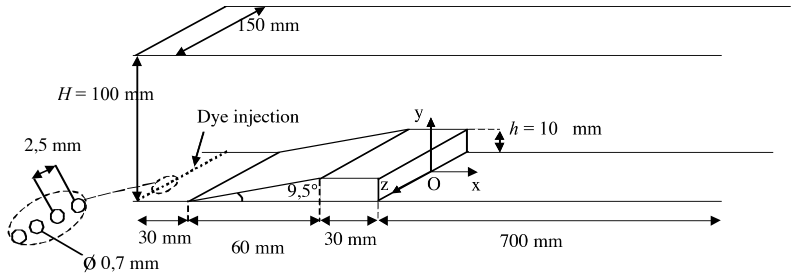

Figure 1. Experimental set-up for the configurations with the 1 cm high step

3. Experimental results

The velocity profile at the step edge, in the plane of symmetry, is not a Poiseuille flow but a flat profile with about a 1 cm thick boundary-layer. In the recirculation zone downstream the step, the velocities are very low compared with those of the mean flow velocity. The separation surface is then submitted to a strong shear. For higher Reynolds numbers, the flow becomes unstationary; the separation surface is subjected to shear-layer instabilities above a critical Reynolds number ($Rc = 313$ for $h = 1$ cm). The present study is only concerned by the stationary regime.

The recirculation length is obtained from the PIV measurements by measuring the distance between the step edge and the point of reattachment on the bottom wall. This point is characterized by a zero longitudinal velocity in the extreme vicinity of the bottom wall. The Figure 2(a) represents the recirculation length in the plane $z = 0$ obtained with the 1 cm high step versus the Reynolds number. In order to quantify the topology of the separated zone, we perform several measurements of the reattachment length in the spanwise direction (corresponding to different PIV planes z = cst).

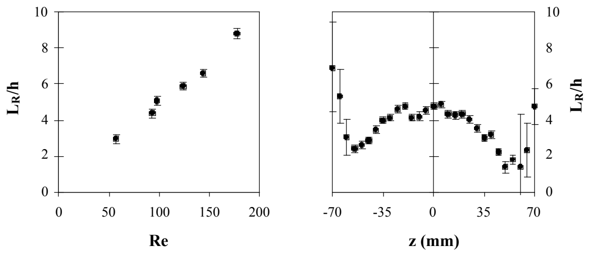

Figure 2. Non-dimensional recirculation length for the 1 cm high step deduced from PIV measurements: (a) measurements in the symmetry plane z =0 versus the Reynolds number; (b) measurements at Re =100 versus z.

The recirculation length versus the spanwise coordinate z is plotted in Figure 2(b). These measurements made at $Re = 100$ indicate a strong three-dimensional effect since the recirculation length ranges from 1.5h to 7h. On both side of the channel ($z < -70$mm and $z > 70$ mm), we observe a side-wall effect similar to the one described by Armaly et al. [1] and later called wall-jets by Williams & Baker [3]. In the centre part of the channel ($-50 < z < 50$), we can observe a spanwise oscillation of the recirculation length with a wavelength of about 3 cm.

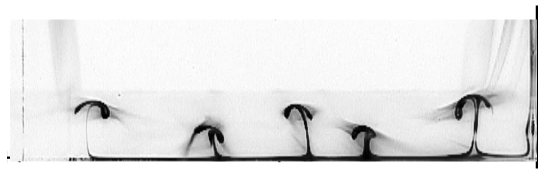

Figure 3. Visualization of the flow in the plane x=25h at Re=100 with the 1 cm high step. The flow is coming in the direction of observation

We performed LIF visualizations in the transversal planes (x= cst). A clear periodic spanwise structure of the flow is observable -Figure 3-. The dye, firstly homogenously injected, separates into five patches for the small step and into five mushroom-like vortices for the big step. These structures reveal the presence of counter-rotating longitudinal structures. We observed this kind of spanwise structures for lower and higher Reynolds numbers in a range of 20 to 200. We were then never able to observe any threshold. Actually, these structures always appear after a very long time (typically 30 minutes) compared to the advection time of the dye to pass above the separated region (1 minute). The disturbance of the velocity field induced by the longitudinal structures should then be very low compared to the basic flow.

The spanwise wavelength corresponds satisfactorily to the spanwise wavelength observed in Figure 2(b) for the reattachment length. We find that the wavelength does not depend on the step height. We checked the influence of the upstream ramp on the flow using an experimental configuration without step: the dye remains homogenously distributed at the bottom wall.

4. Centrifugal instability

We study a possible mechanism for the origin of the three-dimensional structure of the flow. Our strategy is to study the stability of a two-dimensional flow obtained by direct numerical simulation on the same geometry as the experiment..

As the Reynolds numbers are very low, we perform direct numerical simulation DNS of the flow. The numerical procedure is based on a control volume, finite difference method. The equations are solved using the SIMPLE (Semi Implicit Method for Pressure Linked Equation) algorithm with an iterative line-by-line matrix solver. We use a structured mesh with a very fine grid so that it can be used for a rather wide range of Reynolds number. The mesh is refined in the boundary layer regions, in the separation region, and in the recirculation bubble The smallest resolution in the vertical direction is 0.25 mm. The total grid size is 43 000 cells.

We computed a local Rayleigh determinant, corresponding to a local criterion for a potential centrifugal instability as Mutabazi et al.[5] and Sipp & Jacquin [6]. The Rayleigh discriminant Φ , computed numerically from the results of the 2D numerical simulation, is obtained from the expression:

$$\Phi(x,y)=\frac{2U\omega}{R}, \qquad (1)$$

where $U(x,y)$ is the module of the velocity, $\omega(x,y)$ is the vorticity and $R(x,y)$ is the local algebraic radius of curvature that can be expressed [6] from the velocity field as follows:

$$R(x,y)=\frac{U^3}{ua_y-va_x}, \qquad (2)$$

where (u,v) are the components of the velocity field and (a_x,a_y) the components of the convective acceleration $(u.\nabla)u$

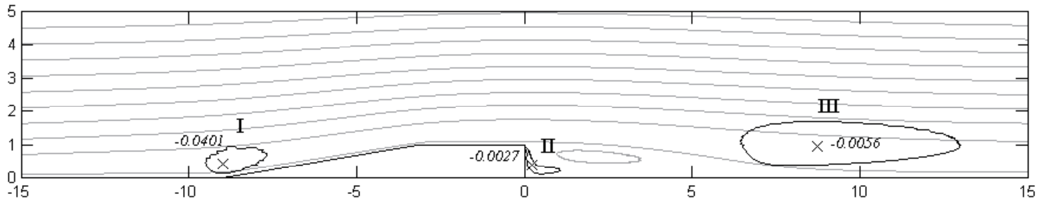

Figure 4. Contour plot of the Rayleigh discriminant Φ (black lines) superimposed with the streamlines obtained from the numerical simulation (grey lines) at Re=100. Three potentially unstable regions appear: each minimum is displayed with a cross and the contour plot around corresponds to its half-minimum value.

The results of the computation are plotted in the Figure 4. We can distinguish three regions of high curvatures where the sign of Φ is negative and the flow at these points(x,y) is potentially unstable: the region in the front of the ramp I, the region in the recirculation zone II and finally the region just above the reattachment location III. The intensity of Φ is measured as the local minima. It is -0.0056 in the region III, -0.0027 in II and -0.0401 in region I. The spatial extension is measured as the contour plot at half the minimum. The largest extension corresponds to region III, the intermediate to region I, the smallest to region II.

The Rayleigh criterion gives a necessary condition for the centrifugal instability but it does not take into account the stabilizing viscosity effect. The Görtler number actually compares the curvature effects with the viscosity effects:

$$G=\frac{U\delta^{3/2}}{\nu R^{1/2}} \qquad (3)$$

where ν is the kinematic viscosity of the fluid and δ is the characteristic size of the unstable zone. When the Görtler number is high enough (above a threshold that has to be defined) the curvature effect dominates the viscosity effects and the flow is unstable.

From the numerical simulation we are able to evaluate the Görtler number G at the three locations of potential instability exhibited in Figure 4. We define the characteristic size of each unstable zone δ as the width of the contour of the half-minimum value of the Rayleigh criterion.

The Figure 5 presents the results of the evaluation of the Görtler number in these three regions versus the Reynolds number. We observe that the largest Görtler number is not found in region I where the Rayleigh criterion is the strongest, but in region III. Moreover, in region I, the Görtler number saturates around 75 while it is still increases in region III up to 400. In region II, the Görtler number remains, in comparison, very small and never exceeds 15.

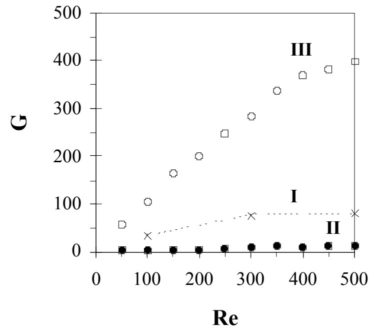

Figure 5. ***Estimated Görtler number for each potentially unstable region versus the Reynolds number (crosses for region I, filled circles for region II and empty circles for region III).***

5. Discussion

A relevant fact is that we do not observe instability when the geometry of the experiment is modified and the step is eliminated. So, the region I, with the ramp alone, is not modified and we can then deduce that region I is stabilized by the viscosity. As a result the value of the Görtler number in the region I is below the threshold of stability. It implies that region II should be stable since the Görtler number is always smaller than this reference value. On the other hand the Görtler number of region III is always larger than the Görtler number of region I, so it is then plausible that region III could be unstable through centrifugal instability.

Our experiment is the first to show a spanwise periodicity of the flow. Previous works [1,3] report sidewall effects but not intrinsic three-dimensional instability. With the support of direct numerical simulation we show that the observed instability is consistent with a centrifugal instability raising just downstream the recirculation zone.

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